ELG 5372 Error Control Coding

Claude D'Amours

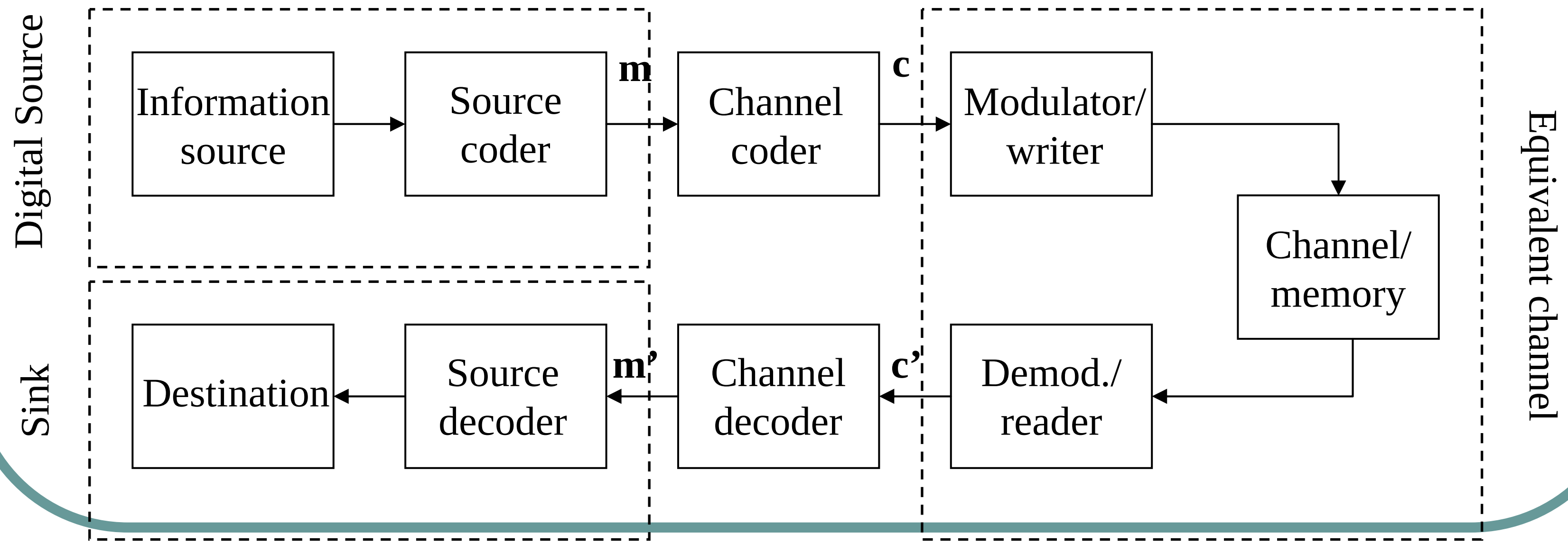
Lecture 1: Introduction to Coding

Introduction

- Shannon demonstrated that by proper encoding of information, errors introduced by a noisy environment can be reduced to any desired level without sacrificing transmission rate, as long as transmission rate is below capacity of channel.
- Since Shannon's work, much research has been done to find efficient encoding and decoding methods.

Introduction (2)

- Transmission and storage of digital information are two processes that transfer data from an information source to a destination.

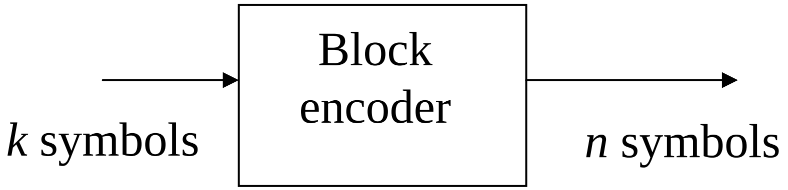


Types of Codes

- Two structurally different types of codes are typically used:
  - Block Codes
    - Hamming
    - BCH, RS
    - LDPC
  - Convolutional Codes
    - Turbo codes typically use convolutional codes as constituent codes
    - TCM based on convolutional codes

Block Codes

- A block of k digital symbols is input to the encoder from which n digital symbols are output (typically $n > k$).



- Each k bit message sequence is mapped to one of M^k possible codewords. Since there are M^n possible M-ary sequences of length n, errors can be detected if an invalid codeword is received.

Block Codes (2)

- Code rate $R = k/n$.
- The message sequence carries k symbols of information.
- The codeword, which carries k symbols of information, is made up of n symbols.
- There are $(n-k)$ redundant symbols.

Convolutional Codes

- A convolutional code also produces *n* symbols for *k* input symbols.
- However, output not only depends on current *k* inputs but also on *km* previous inputs, where *m* is the encoder memory.
- Encoder is implemented by a sequential logic circuit.

Modulation and Coding

- Symbol rate = R_s, signaling interval = $T = 1/R_s$.
- For each symbol, the modulator selects a waveform of duration T to represent the symbol to be transmitted.
- Example BPSK:

$$s_0(t) = \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t + \pi), 0 \le t \le T$$

$$s_1(t) = \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t), 0 \le t \le T$$

Modulation and Coding (2)

- Transmitted signal is:

$$s(t) = \sum_{n=0}^{N} s_i(t - nT)$$

where $i = 0,1,\ldots,M\text{-}1$ and is random ($i = 0$ or 1 for binary case).

- The received signal is:

$$r(t) = a(t)s(t - \tau(t)) + n(t)$$

where $a(t)$ is the time varying channel gain, $\tau(t)$ is the delay introduced by the channel and $n(t)$ is additive noise.

Modulation and Coding (3)

- AWGN Channel
  - $a(t) = a$ and $\tau(t) = \tau$.
- Flat Rayleigh Fading
  - $a(t)$ = time varying Rayleigh envelope
  - $\tau(t)$ introduces time varying phase shift.
- Noise introduces detection errors at the receiver.
- Error rate is function of E_s/N_o.

Modulation and Coding (4)

- BER for BPSK in AWGN is:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

- BER for BPSK in slow flat Rayleigh fading with ideal channel phase compensation:

$$P_b = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right]$$

Modulation and Coding (5)

- Coding increases symbol rate (k info bits without coding, n code bits after coding).
- For same transmitted power, the code bit energy is less than the uncoded bit energy $E_c = REb = (k/n)E_b$.
- Therefore, the probability that a code bit is incorrectly detected is higher than the probability that an uncoded bit is incorrectly detected.

Modulation and Coding (6)

- Coded data streams provide improved bit error rates after decoding due to the error correction capabilities of the code.

Example Hamming (7,4)

0000	0000000	1000	1000110
0001	0001101	1001	1001011
0010	0010111	1010	1010001
0011	0011010	1011	1011100
0100	0100011	1100	1100101
0101	0101110	1101	1101000
0110	0110100	1110	1110010
0111	0111001	1111	1111111

Example Hamming (7,4)

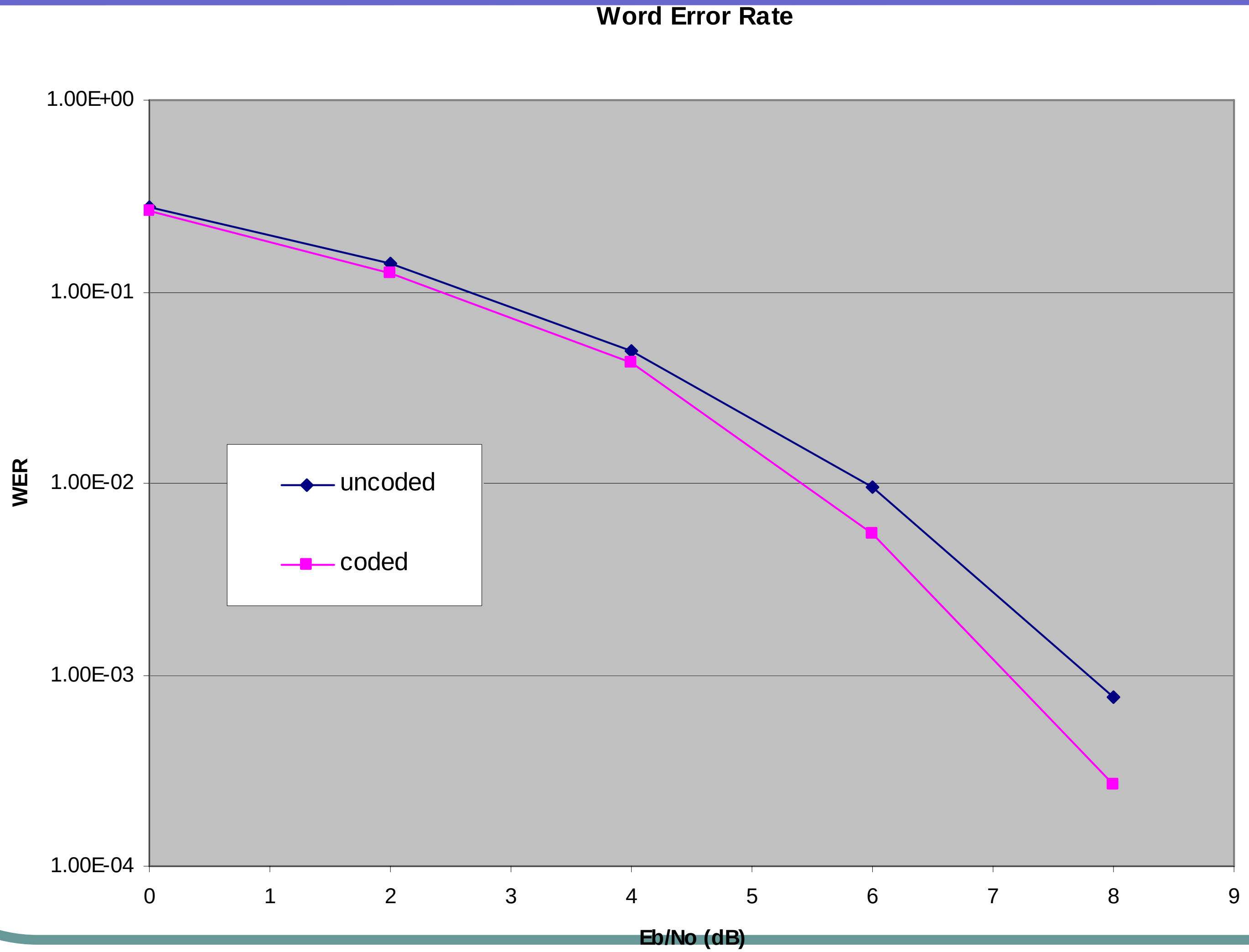
- Assume that we transmit 0000 in the uncoded case.
  - If the first bit is incorrectly detected, we receive 1000, which is a valid message.
- Assume that we transmit 0000000 in the uncoded case
  - If the first bit is detected in error, we receive 1000000, which is not a valid codeword.
  - Error has been detected.
  - Codeword 0000000 differs from 1000000 in only one bit position. All other codewords differ from 1000000 in at least two positions.

Example Hamming (7,4)

- Assuming independent errors
- P(uncoded word error) = $1-(1-P_u)^4$.
- P(coded word error) = $1-(1-P_c)^7-7P_c(1-P_c)^6$.
- In AWGN channel:

$$P_u = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \; P_c = Q\left(\sqrt{\frac{2(4/7)E_b}{N_0}}\right)$$

Example Hamming (7,4) WER



Example Hamming (7,4) BER

- BER
- Uncoded $P_b = P_u$.
- Coded

$$P_b = 9P_c^2(1-P_c)^5 + 19P_c^3(1-P_c)^4 + 16P_c^4(1-P_c)^3 + 12P_c^5(1-P_c)^2 + 7P_c^6(1-P_c)^1 + P_c^7$$

Example Hamming (7,4) BER