ELG3175 Introduction to Communication Systems

Properties of Energy and Power Signals

Energy Signals

- If $x(t)$ is an energy signal with normalized energy E_x :
  - then $y(t) = x(t) \times Ae^{j2\pi f_o t}$ is also an energy signal with $E_y = A^2 E_x$;
  - And $z(t) = x(t) \times A\cos 2\pi f_o t$ is also an energy signal with $E_z = (A^2/2)E_x$; (this assumes that $f_o > B_x$ where B_x is the bandwidth of $x(t)$)

- $Y(f) = AX(f-f_o)$. Donc $G_y(f) = |Y(f)|^2 = |AX(f-f_o)|^2 = A^2 Gx(f-f_o)$. E_y est donnée par :

$$E_y = A^2 \int_{-\infty}^{\infty} G_x(f - f_o)\,df$$

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Energy Signals

- Replace f-f_o by f' and we obtain :

$$E_y = A^2 \int_{-\infty}^{\infty} G_x(f')df' = A^2 E_x$$

- For $z(t) = x(t) \times A\cos 2\pi f_o t$, we note that z(t) can be expressed as:

$$z(t) = \tfrac{A}{2} x(t) e^{j2\pi f_o t} + \tfrac{A}{2} x(t) e^{-j2\pi f_o t}$$

Power signals

- Similarly, we can show that if $x(t)$ is a power signal with normalized power P_x :
  - Then $y(t) = Ae^{j2\pi f_o t}x(t)$ is also a power signal with power $P_y = A^2P_x$;
  - And $z(t) = x(t)\times A\cos 2\pi f_o t$ is also a power signal with power $P_z = (A^2/2)P_x$; (for $f_o > B_x$ where B_x is the bandwidth of $x(t)$).

Symmetry of autocorrelation functions

- If $x(t)$ is a real signal, its autocorrelation function is an even function.
- Assume that $x(t)$ is an energy signal and that $x(t) = x^*(t)$, $\varphi_x(\tau)$ is given by :

$$\varphi_x(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt$$

- Remplaçons $t-\tau$ par t' et on obtient :

$$\varphi_x(\tau) = \int_{-\infty}^{\infty} x(t'+\tau)x^*(t')dt' = \int_{-\infty}^{\infty} x(t'+\tau)x(t')dt' = \varphi_x(-\tau)$$

- Similarly, we can show that $R_x(\tau) = R_x(-\tau)$ if $x(t)$ is a real power signal.

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Symmetry of Energy and Power Spectral Densities

- If $x(t)$ is a real signal, then its ESD or PSD is an even function.
  - Since x(t) is real, its autocorrelation is even.
  - The ESD or PSD is the Fourier Transform of the ESD or PSD.
  - The FT of an even function is always even.

Symmetry of Energy and Power Spectral Densities

- Assume that $x(t)$ is a real power signal with autocorrelation function $R_x(\tau)$.
- Its PSD is :

$$S_X(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

$$S_x(-f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi(-f)\tau} d\tau$$

- Replace τ by $-u$ and we obtain

$$S_x(-f) = -\int_{\infty}^{-\infty} R_x(-u) e^{-j2\pi(-f)(-u)} du \quad = \int_{-\infty}^{\infty} R_x(u) e^{-j2\pi(f)(u)} du = S_x(f)$$

Multiplication by $\cos(2\pi f_o t)$

- Suppose that $x(t)$ is an energy signal and that $y(t) = Ax(t)\cos(2\pi f_o t)$ (for $f_o > B_x$ where B_x is the bandwidth of $x(t)$).
- The autocorrelation function of $y(t)$ is :

$$\varphi_y(\tau) = A^2 \int_{-\infty}^{\infty} x(t)x^*(t-\tau)\cos(2\pi f_o t)\cos(2\pi f_o (t-\tau))dt$$

$$= \frac{A^2}{2}\int_{-\infty}^{\infty} x(t)x^*(t-\tau)\cos(2\pi f_o \tau)dt + \frac{A^2}{2}\int_{-\infty}^{\infty} x(t)x^*(t-\tau)\cos(2\pi f_o (2t-\tau))dt$$

$$= \frac{A^2}{2}\cos(2\pi f_o \tau)\int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt = \frac{A^2}{2}\varphi_x(\tau)\cos(2\pi f_o \tau)$$

Multiplication by cos($2\pi f_o t$)

- Similarly, if $x(t)$ is a power signal then the autocorrelation of $y(t) = Ax(t)\cos(2\pi f_o t)$ is :

$$R_y(\tau) = \frac{A^2}{2} R_x(\tau)\cos(2\pi f_o \tau)$$

- (for $f_o > B_x$ where B_x is the bandwidth of $x(t)$).
- Therefore $G_y(f) = (A^2/4)G_x(f-f_o)+(A^2/4)G_x(f+f_o)$ if $x(t)$ is an energy signal and $S_y(f)= (A^2/4)S_x(f-f_o)+(A^2/4)S_x(f+f_o)$ if $x(t)$ is a power signal.

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