ELG3175 Introduction to Communication Systems

Digital Communication Example and Introduction to Information theory

Digital Communication Engineering Example

- The communication engineer must design a communication system that meets certain criteria
  - Data rate determined by SQNR (in ADC)
  - Bandwidth considerations

Example

- An analog signal, $m(t)$, has the following properties:
  - $-5V < m(t) < 5V$
  - Bandwidth = 10 kHz
  - Power = 4W
- We wish to convert this to a digital signal using PCM and then transmit using digital PAM.
- Design constraints
  - SQNR > 70 dB
  - Bandwidth < 50 kHz
  - Raised cosine pulses with 30% excess bandwidth is used.
  - Sampling rate is 30% greater than Nyquist rate.

What we need to solve

- What is the minimum number of quantization levels required if we want SQNR > 70 dB?
- If we represent each sample by N bits/sample, what should N be?
- If we sample at a sample rate $f_s = 1.3f_N$, what is the data rate in bits/sec at the output of the PCM modulator?
- What is minimum value of M for M-ary PAM to satisfy our bandwidth constraint?

SQNR

- $SQNR = 12P_m/\Delta^2 = 3P_mL^2/m_p{}^2 > 10^7$.
- $L^2 > 10^7 25/(3)(4) = 2.08\times10^7$.
- $L > 4564$
- $L = 2^N$, therefore $N = \log_2(L) > \log_2(4564)$
- $N>12.2$ bits/sample, which means we would select $N = 13$ bits/sample.

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Bandwidth

- f_N = (2)(10 kHz) = 20 ksamples/sec
- $f_s = 1.3f_N$ = 26 ksamples/sec
- $R_b = f_s N$ = 338 kbps
- $BW = R_s(1+\alpha)/2$
- $R_s < 45\text{kHz}\times 2/(1+\alpha)$
- $R_s < 90/1.3 = 69.23$ ksymbols/sec
- $R_s = R_b/k < 69.23$ ksymbols/sec
- $k > R_b/69.23 = 4.88$
- We choose k = 5, M = 2k, therefore we should use 32-level PAM

Information theory

- Information theory quantifies information content and determines the limits on the amount of information we can transmit
- Claude Shannon quantified information in 1949.
- Huffmann coding allows us to transmit data efficiently
- Error Control coding allows us to use controlled redundancy to correct errors that occur in transmission.

Amount of information contained in a message

- We have a source that outputs messages from a set of possible messages $\{m_1, m_2, \ldots, m_N\}$.
- Let's assume that the source is memoryless which means that the transmission of future messages does not depend on which messages were transmitted in the past. (messages are transmitted independently)
- Each message has a probability of being transmitted $\{p(m_1), p(m_2), \ldots p(m_N)\}$.
- Messages with low probabilities of transmission have higher information content.
  - Example: m_1 = it will be sunny, m_2 = there will be a tornado.

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Amount of information

- Here $p(m_1)>p(m_2)$ but $I(m_1)<I(m_2)$.
- Suppose message $m_3 = m_1m_2$. Assuming independent messaging, $p(m_3) = p(m_1)p(m_2)$, but clearly $I(m_3) = I(m_1)+I(m_2)$.
- Therefore $I(m_i) = \log_b(1/p(m_i)) = -\log_b(p(m_i))$.
- $b = 2$, $I(m_i)$ is measured in bits (or Shannons).
- $b = 3$, $I(m_i)$ is measured in ternary symbols
- $b = e$, $I(m_i)$ is measured in nats
- $b = 10$, $I(m_i)$ is measured in Hartleys

Example

- M = $\{m_1, m_2, m_3, m_4\}$ with P = {0.35, 0.11, 0.45, 0.09}
- $I(m_1)$ = 1.51 bits
- $I(m_2)$ = 3.18 bits
- $I(m_3)$ = 1.15 bits
- $I(m_4)$ = 3.47 bits

- The probability that the source outputs m_1 followed by message m_2 is $0.35 \times 0.11 = 0.0385$. The information in these two messages is $-\log_2(0.0385) = 4.69$ bits = 1.51+3.18.

Average information: Entropy of the source

- At each signaling instant, the source outputs a message.
- Assuming that the message output is m_i, then the source has output $I(m_i)$ bits of information.
- The expected amount of information to be output at each signaling instant is called the entropy of the source $H(M)$.

$$H(M) = -\sum_{i=1}^{N} p(m_i) \log_b p(m_i)$$

Example

- M = $\{m_1, m_2, m_3, m_4\}$ et P = {0.35, 0.11, 0.45, 0.09}
- $I(m_1)$ = 1.51 bits
- $I(m_2)$ = 3.18 bits
- $I(m_3)$ = 1.15 bits
- $I(m_4)$ = 3.47 bits
- $H(M)$ = 0.35(1.51)+0.11(3.18)+0.45(1.15)+0.09(3.47) = 1.71 bits/message.

Source coding

- Consider the source of the previous example.
- $H(M)$ = 1.71 bits/message.
- If we encode the 4 messages as follows, m_1 = 00, m_2 = 01, m_3 = 10 et m_4 = 11, we get an average message length of 2 bits/message. L is the average message length.
- We can show that $H(M) \leq L$.
- Suppose we encode the source as follows:
- m_1 = 01, m_2 = 000, m_3 = 1 and m_4 = 001. L = 0.45(1)+0.35(2)+0.11(3)+0.09(3) = 1.75 bits/message.
- Example: 0101010000100100100011 = m_1, m_1, m_1, m_2, m_1, m_4, m_4, m_4, m_4, m_3 : it is uniquely decodable

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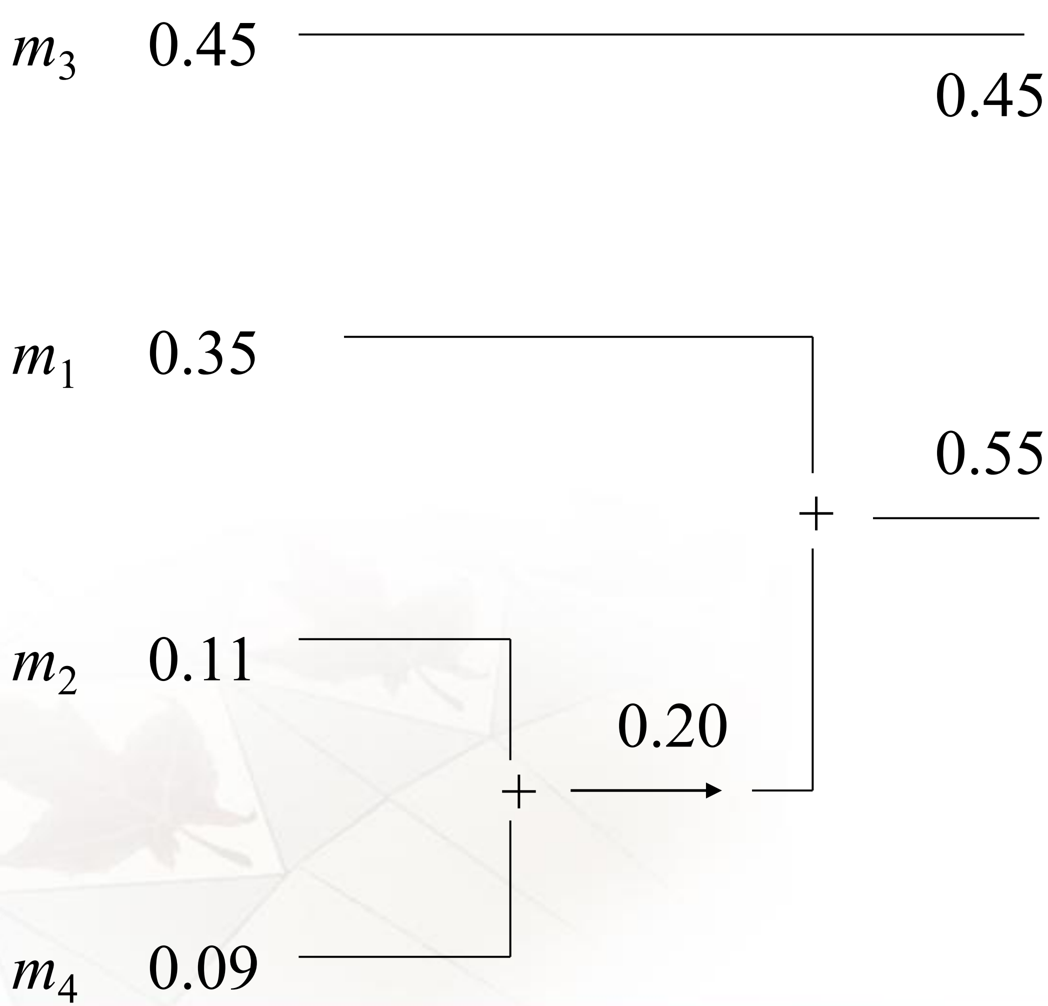
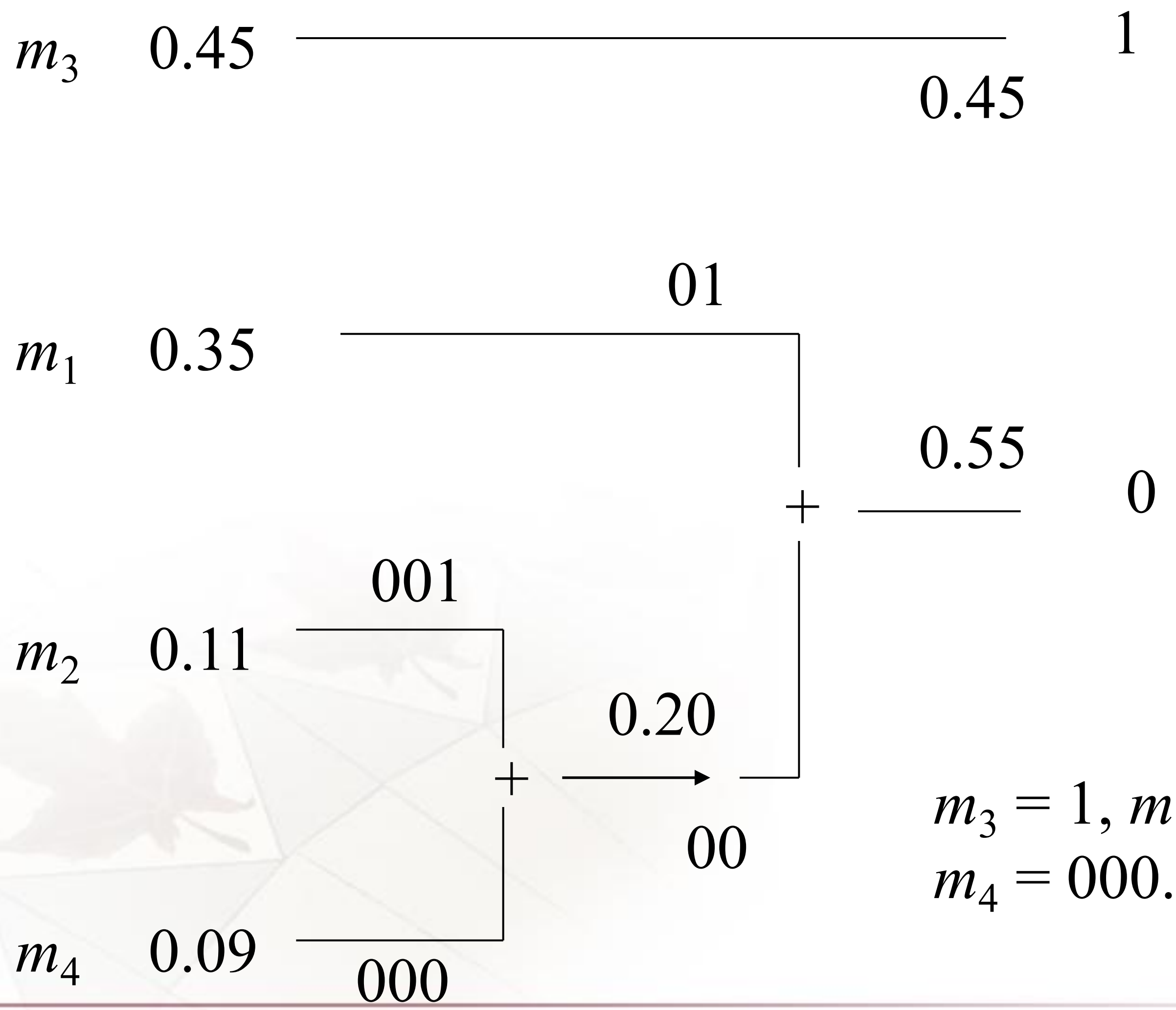
Source coding

- Supose we encode the source as follows:
- m_1 = 10, m_2 = 100, m_3 = 1 and m_4 = 010. L = 0.45(1)+0.35(2)+0.11(3)+0.09(3) = 1.75 bits/message.
- Example: 1010 = m_1, m_1, or m_3, m_4. It is not uniquely decodable.
- The first one is uniquely decodable because it is a prefix conditionned code. This means that the prefix of any codeword is not another codeword.

Huffmann codes

- We can use Huffmann’s coding algorithm to design prefix conditionned codes.


m_3 0.45
0.45
m_1 0.35
0.55
+
m_2 0.11
0.20
+
m_4 0.09



$m_3 = 1$, $m_1 = 01$, $m_2 = 001$
$m_4 = 000$.