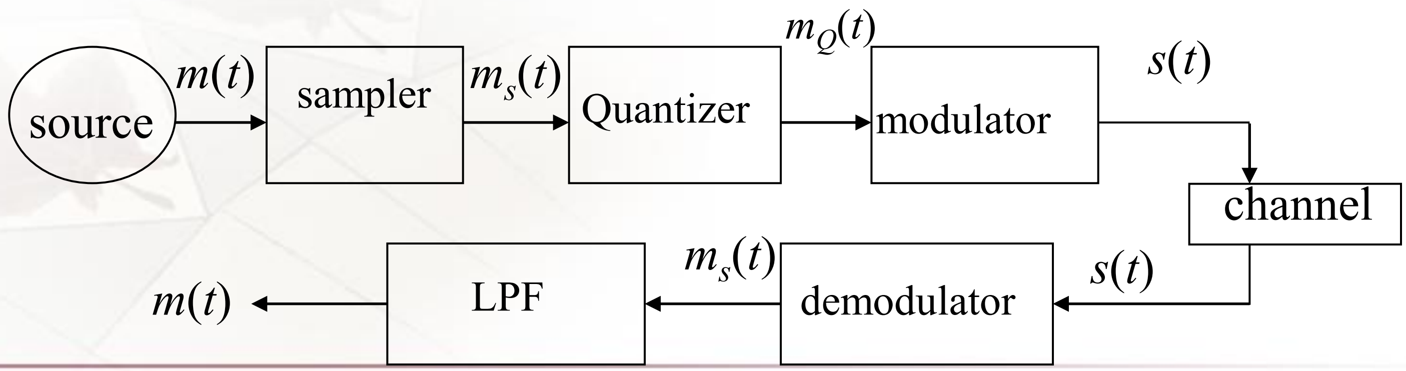
ELG3175 Introduction to Communication Systems

Digital Communications: Analog to Digital Conversion, PAM, PWM and PCM

Analog to Digital Conversion

- MP3, CDs, 2^{nd}, 3rd and 4th generation cellular, satellite radio, Bell Expressvu etc.
  - Technologies that transmit audio and video signals digitally.
  - Digital signals are more robust in the presence of noise and interference compared to analog signals.
  - The first step in ADC is sampling.



Sampling theory

- Let us start with an analog signal $m(t)$ that has bandwidth B_m.
  - We wish to sample this signal so as to produce a digital signal.
  - We represent the sampled signal $m_s(t)$ by:

$$m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) = m(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

- where $T_s = 1/f_s$ is the sampling interval and f_s is the sampling rate. The spectum of $m_s(t)$ is:

$$M_s(f) = M(f) * \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right\}$$

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Spectrum of a sampled signal

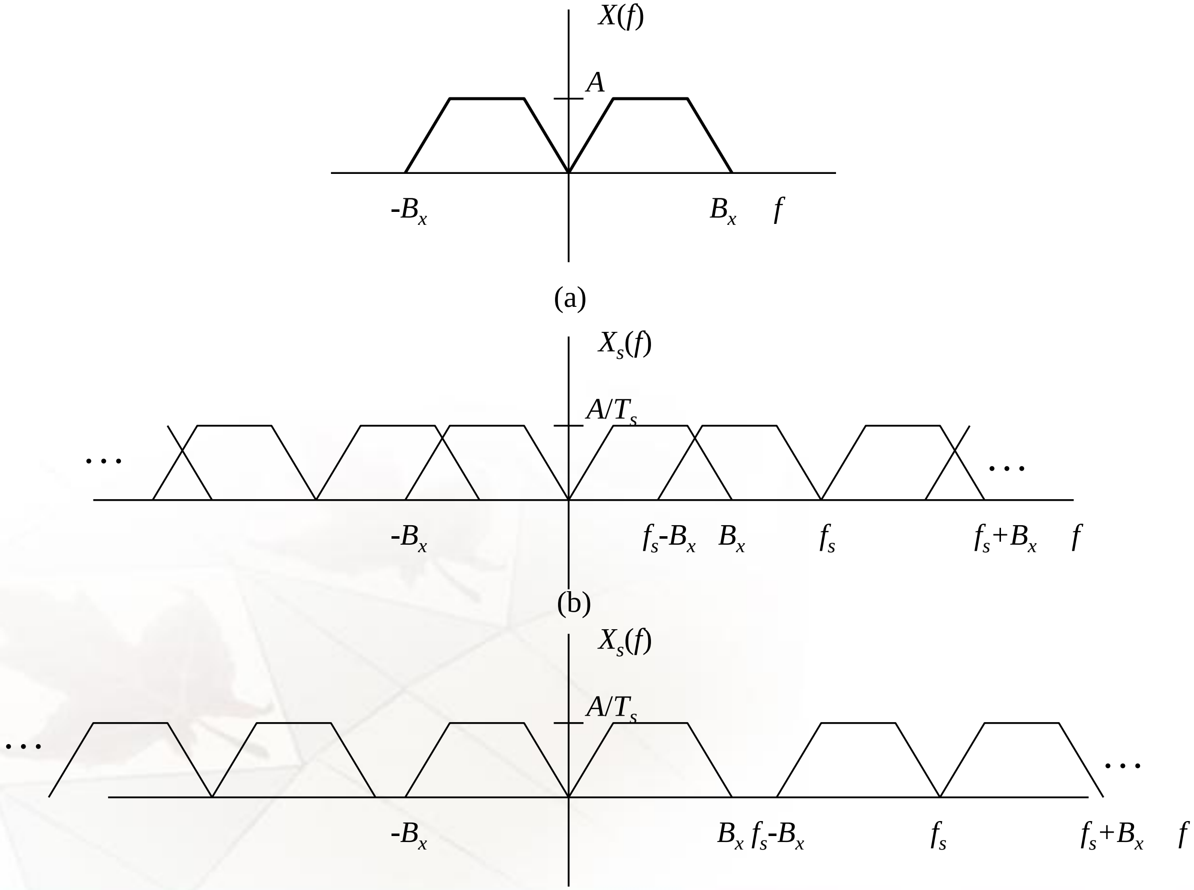
- The signal $\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$ is periodic with period T_s.
  - We can represent it by a Fourier series.

$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi n f_s t} = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{j2\pi n f_s t}$$

where $X_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) e^{-j2\pi n f_s t} dt = \frac{1}{T_s}$

therefore $\mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \right\} = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$ and…

$$M_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} M(f - nf_s)$$



Reconstruction of $m(t)$ from $m_s(t)$

- $M_s(f)$ is given in the previous figure for $f_s < 2B_m$ (b) and $f_s > 2B_m$ (c).
- We can get $M(f)$ from $M_s(f)$ by using a LPF in (c).
- Therefore, to be able to recover $m(t)$ from $m_s(t)$, we need $f_s > 2B_m$. The Nyquist rate is $f_s = 2B_m$.

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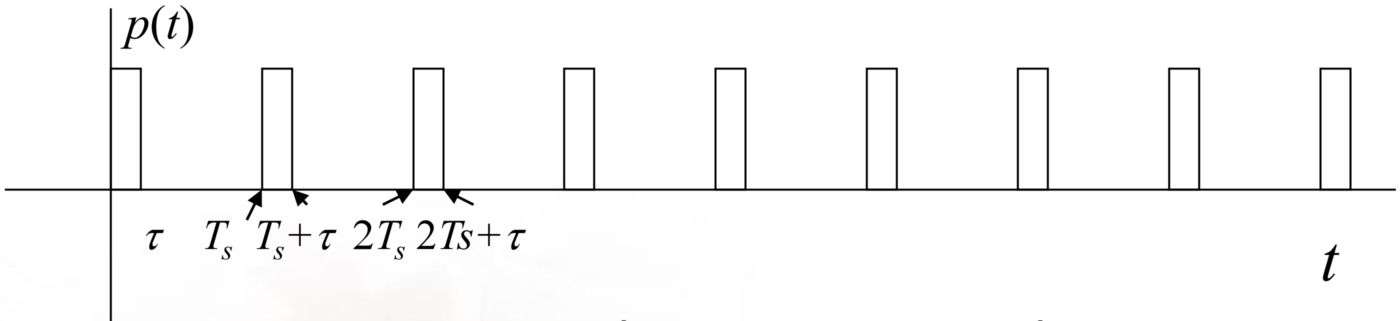
Periodic impulse train

- The impulse train $x(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$ is not a practical signal.
- In reality a practical sampling signal is given by:



- The signal p(t) is $$p(t)=\sum_{n=-\infty}^{\infty}g(t-nT_s)=g(t)*\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$$
- and $$P(f)=\frac{G(f)}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$
- Therefore $B_p = B_g$.

Exemple

- In the previous example, $g(t) = \Pi[(t-\tau/2)/\tau]$, therefore $G(f) = \tau\text{sinc}(f\tau)e^{-j\pi f\tau}$.
- Therefore

$$P(f) = \frac{\tau}{T_s}\text{sinc}(f\tau)e^{-j\pi f\tau}\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$= \frac{\tau}{T_s}\sum_{n=-\infty}^{\infty}\text{sinc}(nf_s\tau)e^{-j\pi nf_s\tau}\delta(f-nf_s)$$

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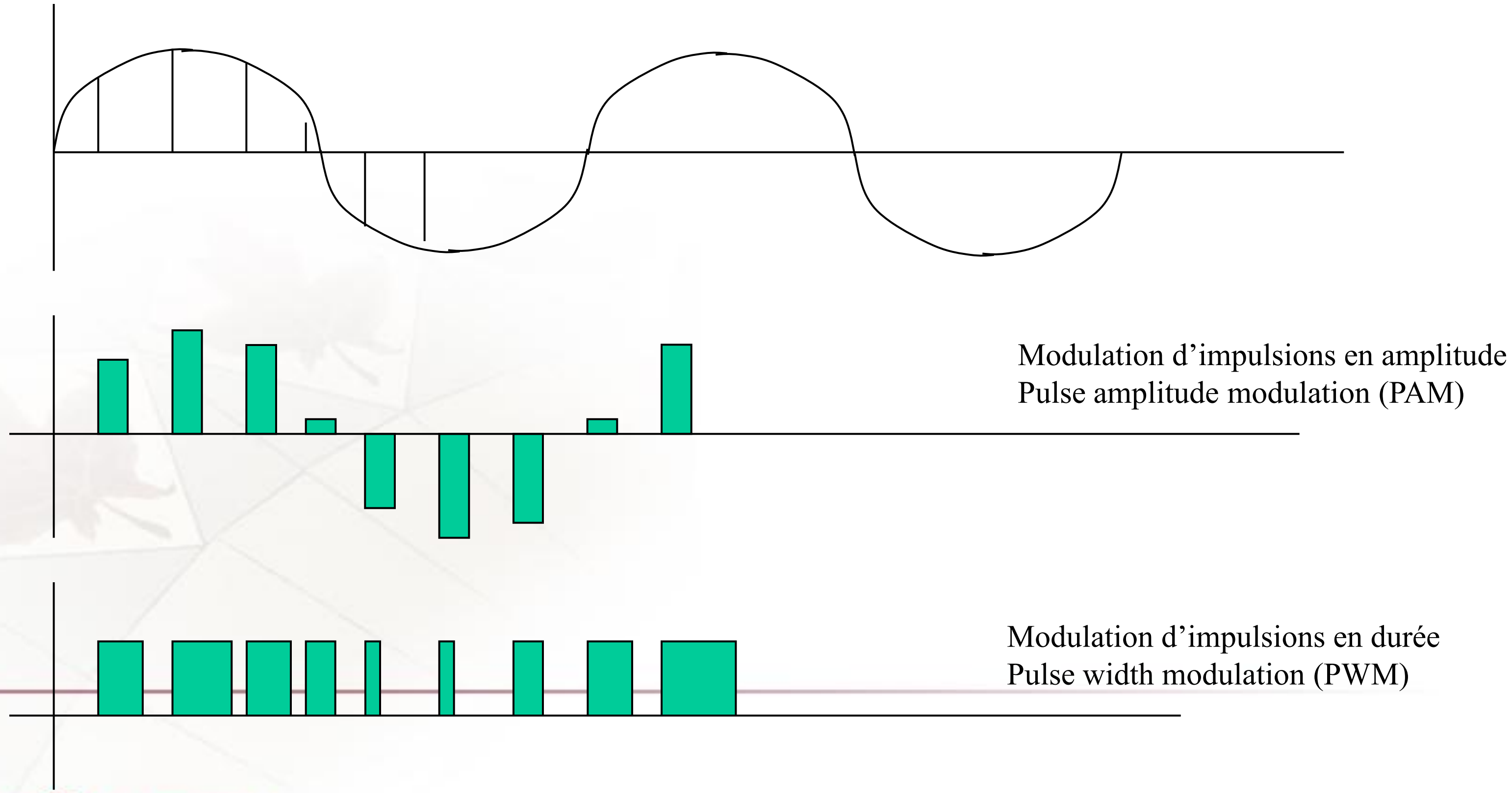
Pulse modulation

- We can transmit samples of a signal using impulses.



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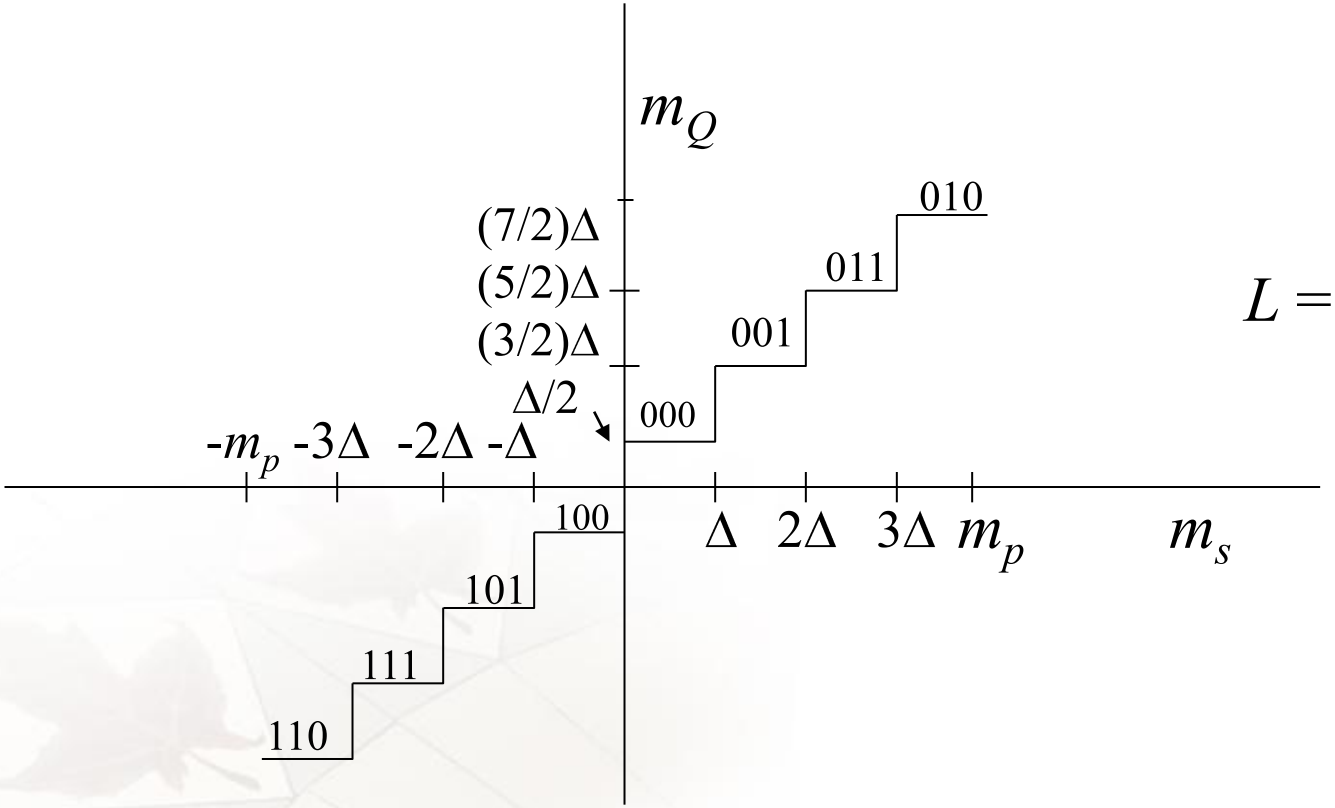
Pulse coded modulation (PCM)

- We want to convert each sample of m_s(t) into a codeword of length N bits.
- Assuming that $-m_p$ < m(t) <$+m_p$ any sample ms(nTs) can take on an infinite number of values between the maximum and minimum value of m(t).
- But a codeword of length N can only represent 2^N different values.
- We need to quantize (round) the samples prior to encoding.

Input-output relationship of a uniform quantizer



$L = 2^N$ levels

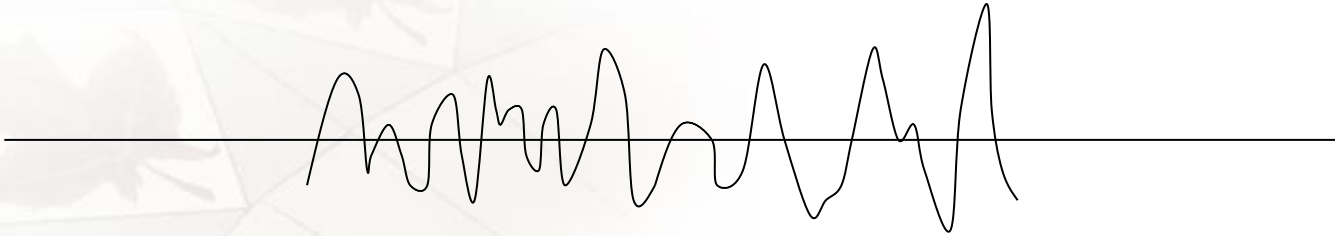
010101001000011 = (7/2)Δ, -(3/2)Δ, (3/2)Δ, Δ, (5/2)Δ.

Quantization noise

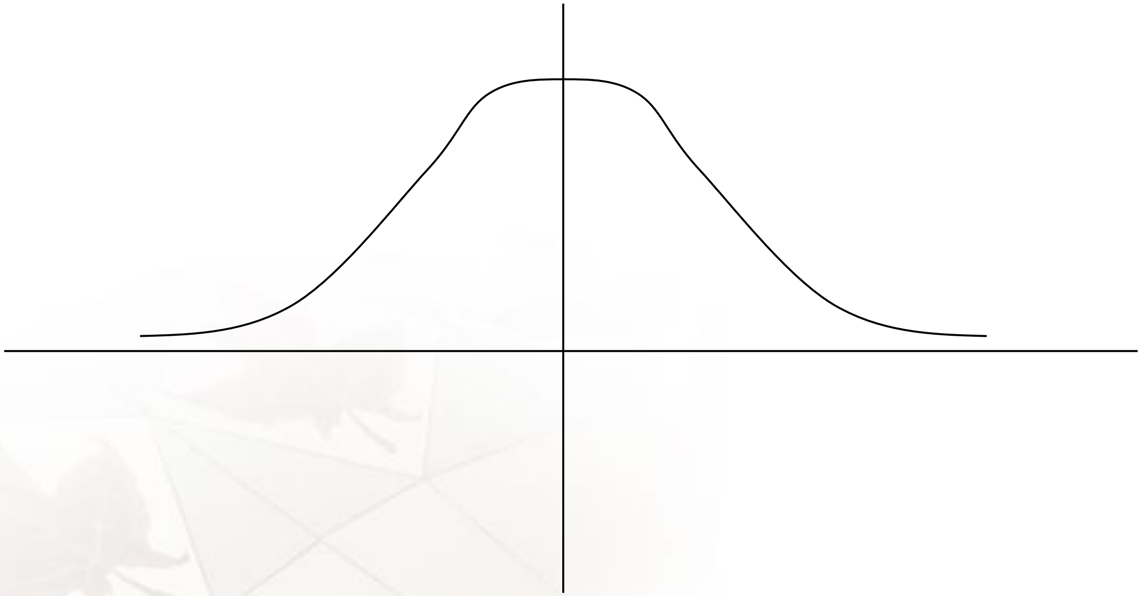
- $m_Q(nT_s) = m_s(nT_s)+e_Q(nT_s)$.
- $e_Q(nT_s) = m_Q(nT_s) - m_s(nT_s)$
- $-\Delta/2 < e_Q(nT_s) < \Delta/2$
- When there are many quantizer levels, we can assume that the error is uniformly distributed between $-\Delta/2$ et $\Delta/2$.
- $f_e(x) = 1/\Delta$ for $-\Delta/2 < x < \Delta/2$. (and 0 otherwise).
- The power of a random signal is $E[e_Q{}^2(nT_s)] = \Delta^2/12$.
- $L\Delta = 2m_p$. ($L = 2^N$). Therefore $\Delta = 2m_p/L$. Therefore the quantization noise is $\Delta^2/12 = m_p{}^2/3L^2$.
- Signal to quantization noise ratio SQNR $= 3L^2P_m/m_p{}^2$.

SQNR

- SQNR is proportional to P_m.
- Power depends on amplitude (volume) (exemple: some people speak louder than others). Large variation between samples, 40dB.
- Most samples are closer to 0 than to peak values.

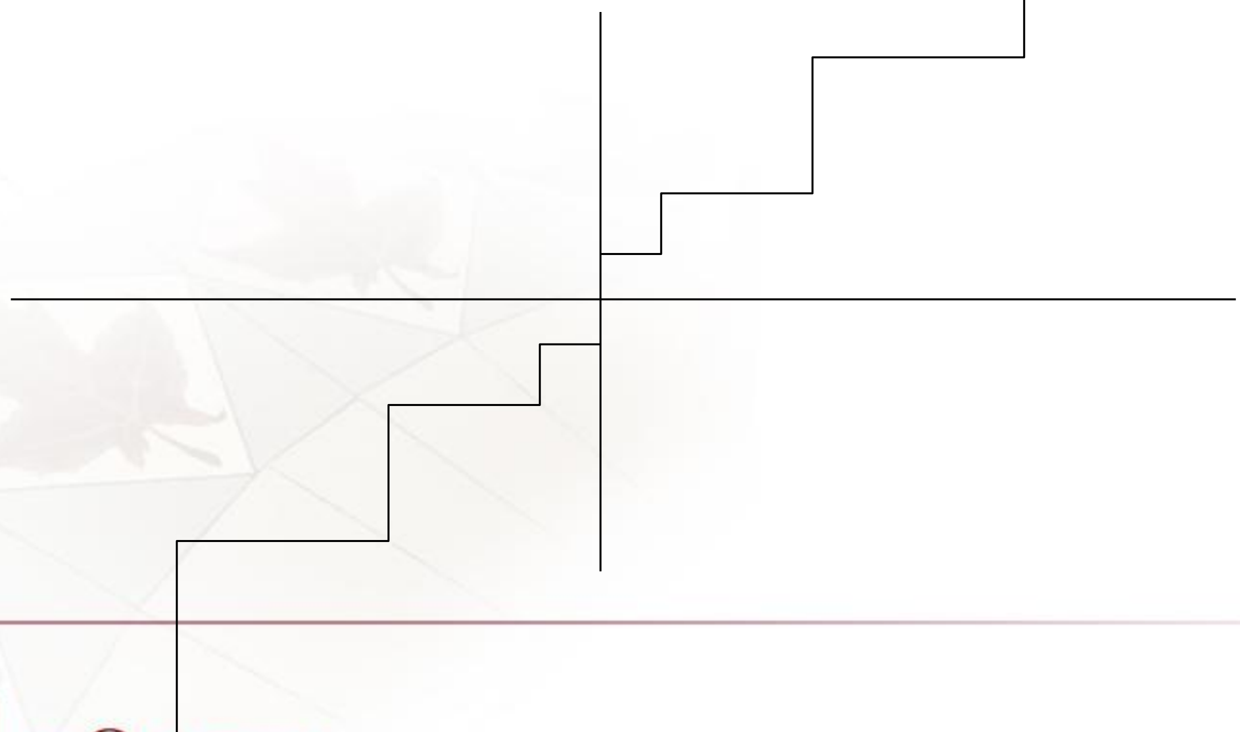


Pdf of a voice signal



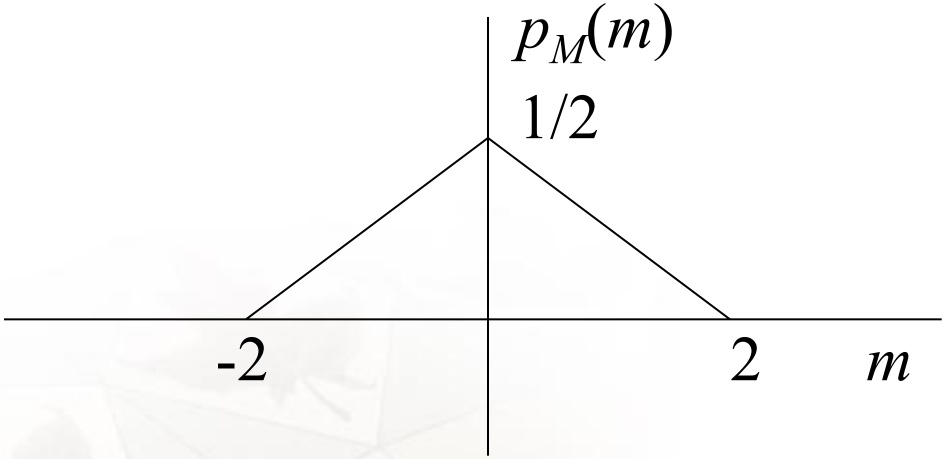
Power of the quantization error

$$E\left[e_Q^2(nT_s)\right] = \sum_{i=1}^{L} \frac{\Delta_i^2}{12} P\left(m(nT_s) \in R_i\right)$$



Example

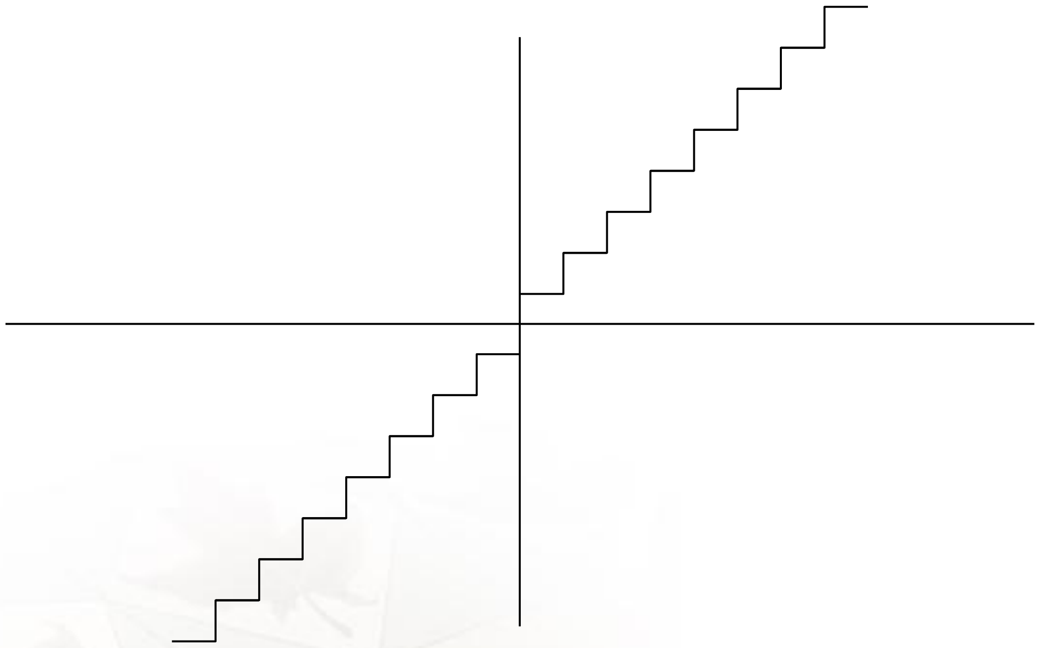
- Uniform vs nonuniform quantization for a random voice signal



$E[m] = 0$

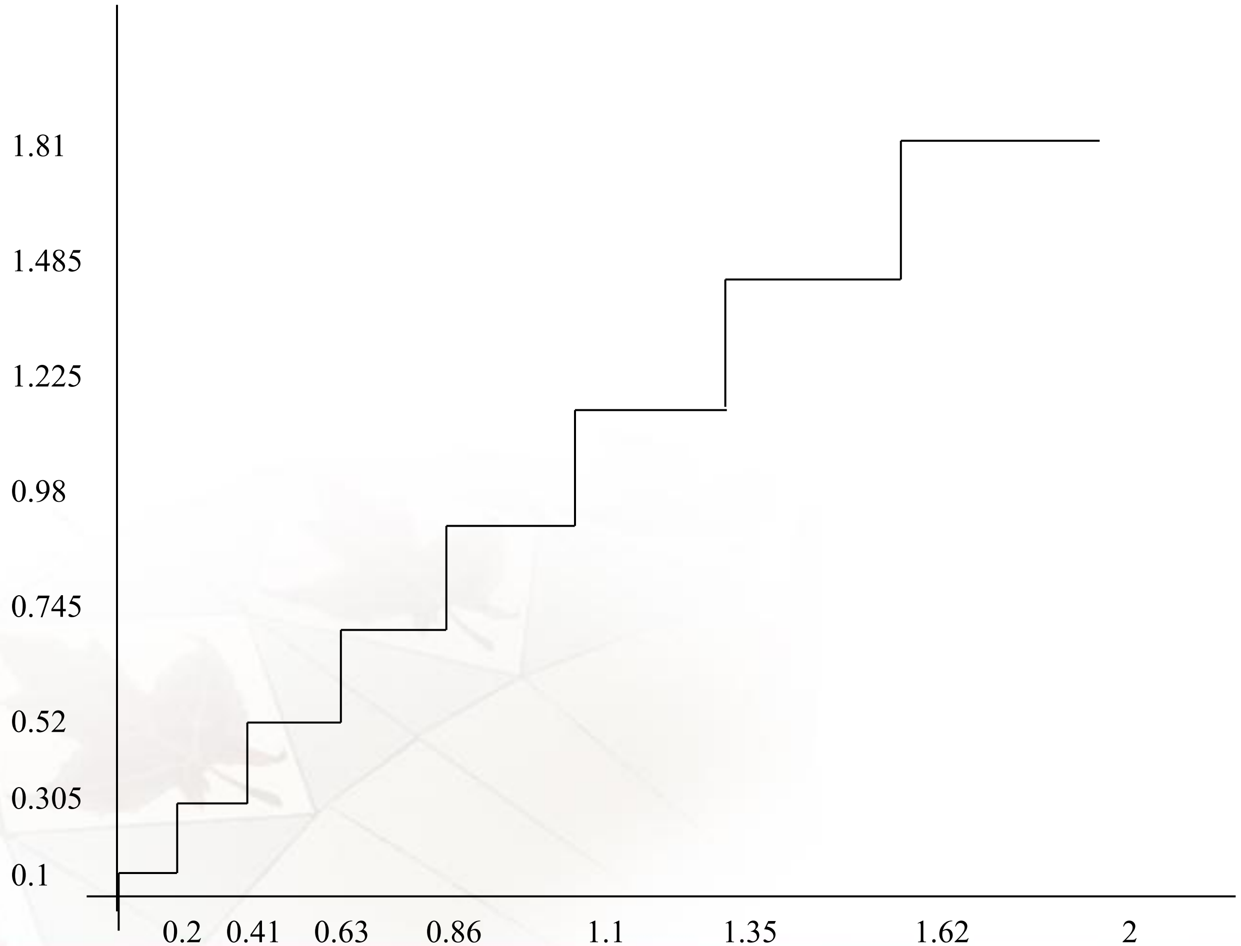
$E[m^2] = 2/3$

4 bit uniform quantization



$\Delta = ¼$. $E[e_Q^2(T_s)] = 1/(16\times12) = 1/192$. SQNR = 128 = 21 dB.

16 bit non-uniform quantization

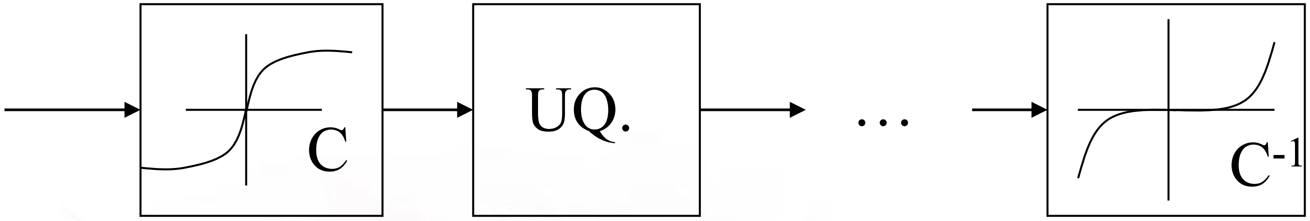


- P(0<m<0.2) = 0.095, Δ = 0.2 (same for P(-0.2<m<0))
- P(0.2<m<0.41) = 0.089 , Δ = 0.21
- P(0.41<m<0.63)=0.081, Δ = 0.22
- P(0.63<m<0.86) = 0.07 , Δ = 0.23
- P(0.86<m<1.1)= 0.61 , Δ = 0.24
- P(1.1<m<1.35)=0.048 , Δ = 0.25
- P(1.35<m<1.62)=0.035 , Δ = 0.27
- P(1.62<m<2)=0.018 , Δ = 0.28

$E[e_Q^2(T_s)] = 2\times[0.095\times 0.2^2/12 + 0.089\times 0.21^2/12 + 0.081\times 0.22^2/12 + \ldots$
$= 1/232.3$. SQNR = 154.8 = 21.9 dB

Compression/Expander: Compandered

- "Compresser – Expander" = "compander"



Produces non-uniform quantization.