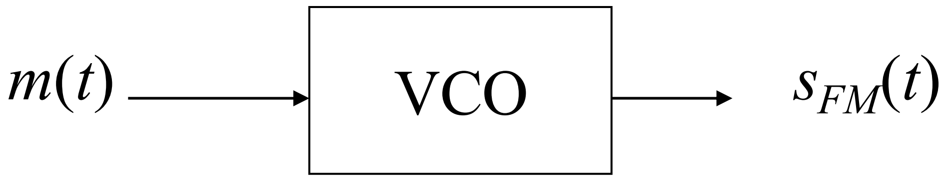
ELG3175 Introduction to Communication Systems

Modulation and Demodulation of FM signals

Generation of WBFM Signals

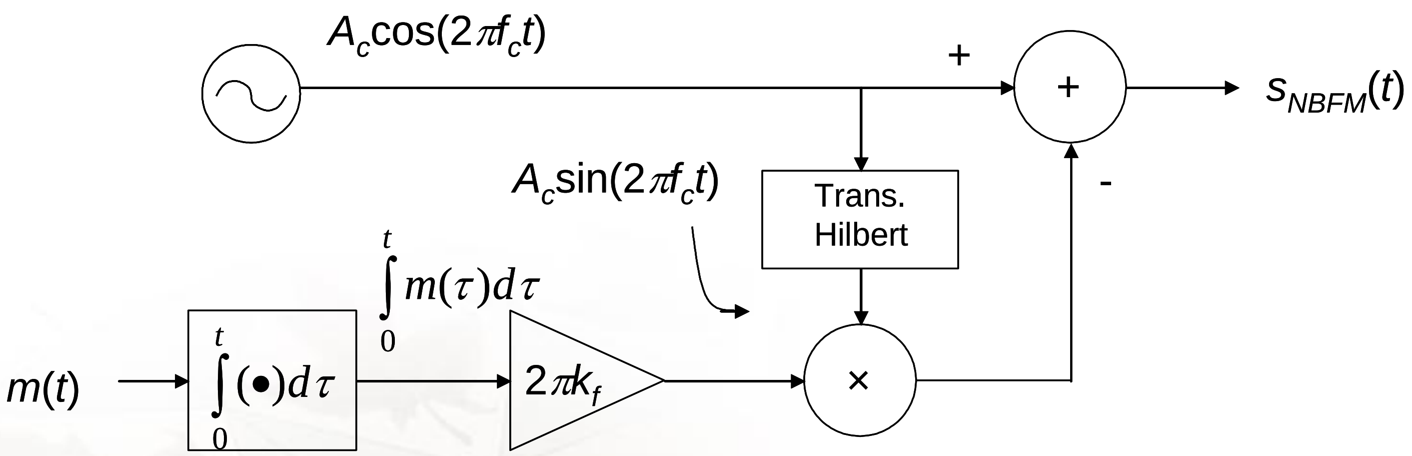
- Direct method
  - Voltage Controlled Oscillator (VCO)



- Indirect method
  - Armstrong's method

Armstrong's Method

- NBFM Modulator



Armstrong's method

- Nonlinearity
  - $v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$
  - $v_i(t) = s_{NBFM}(t)$.
  - Example $s_{NBFM}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t)dt) = A_c \cos(\theta_i(t))$.
  - $v_o(t) = a_1 s_{NBFM}(t) + a_2 s^2_{NBFM}(t) + a_3 s^3_{NBFM}(t) \ldots$
  - $v_o(t) = a_1 A_c \cos(\theta_i(t)) + a_2 A_c^2 \cos^2(\theta_i(t)) + a_3 A_c^3 \cos^3(\theta_i(t)) \ldots$
  - $v_o(t) = a_1 A_c \cos(\theta_i(t)) + a_2 A_c^2/2 + (a_2 A_c^2/2)\cos(2\theta_i(t)) + (3a_3 A_c^3/4)\cos(\theta_i(t)) + (a_3 A_c^3/4)\cos(3\theta_i(t)) \ldots$
  - $n\theta_i(t) = 2\pi(nf_c)t + 2\pi(nk_f)\int m(t)dt$ (carrier frequency = nf_c and $k_f' = nk_f$ therefore $\beta_F' = n\beta_F$).
- BPF is used to pass the spectral component centred @ f = nf_c.

Demodulation of FM signals

- Differentiator plus envelope detection
- Frequency discriminator.
- Frequency counter.

Differentiator and envelope detector

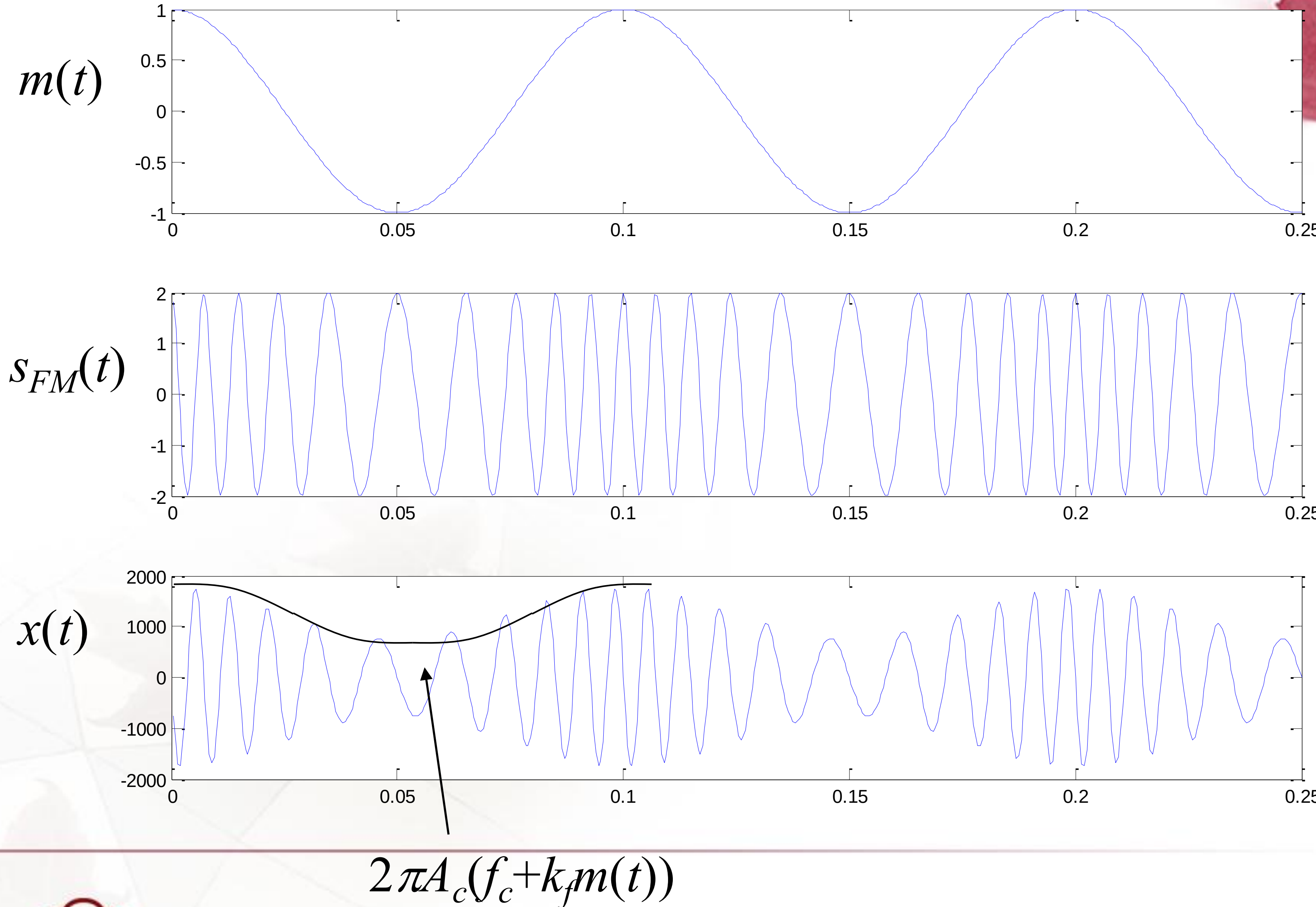
Differentiator and envelope detector

$$
\begin{aligned}
x(t) &= \frac{ds_{FM}(t)}{dt} \\
&= \frac{d}{dt}\left(A_c \cos\left(\theta_i(t)\right)\right) \\
&= -\frac{d\theta_i(t)}{dt} A_c \sin\left(\theta_i(t)\right) \\
&= 2\pi A_c f_i(t) \sin\left(2\pi f_c t + 2\pi k_f \int m(t)dt + \pi\right) \\
&= 2\pi A_c \left(f_c + k_f m(t)\right) \sin\left(2\pi f_c t + 2\pi k_f \int m(t)dt + \pi\right)
\end{aligned}
$$

$f_c >> |k_f m(t)|$ alors $2\pi A_c(f_c + k_f m(t)) > 0$.

Example

- $m(t)$ = cos2π10t, f_c = 100, A_c = 2, k_f = 40 Hz/V.
- $s_{FM}(t)$ = 2cos(2π200t+4sin2π10t)
- $x(t)$ = 4π(100+40cos2π10t)sin(2π100t+4sin2π10t+π)



$m(t)$
$s_{FM}(t)$
$x(t)$
1
0.5
0
-0.5
-1
2
1
0
-1
-2
2000
1000
0
-1000
-2000
0
0.05
0.1
0.15
0.2
0.25
$2\pi A_c(f_c+k_f m(t))$

Differentiator and envelope detector

- Output of envelope detector
  - $y(t) = 2\pi A_c(f_c+k_f m(t)) = 2\pi A_c f_c + 2\pi A_c k_f m(t)$
  - Assuming that m(t) has no DC component ($M(f) = 0$ for $f = 0$), then
- Output of DC block
  - $z(t) = 2\pi A_c k_f m(t) = Km(t)$.

Fluctuations in received amplitude

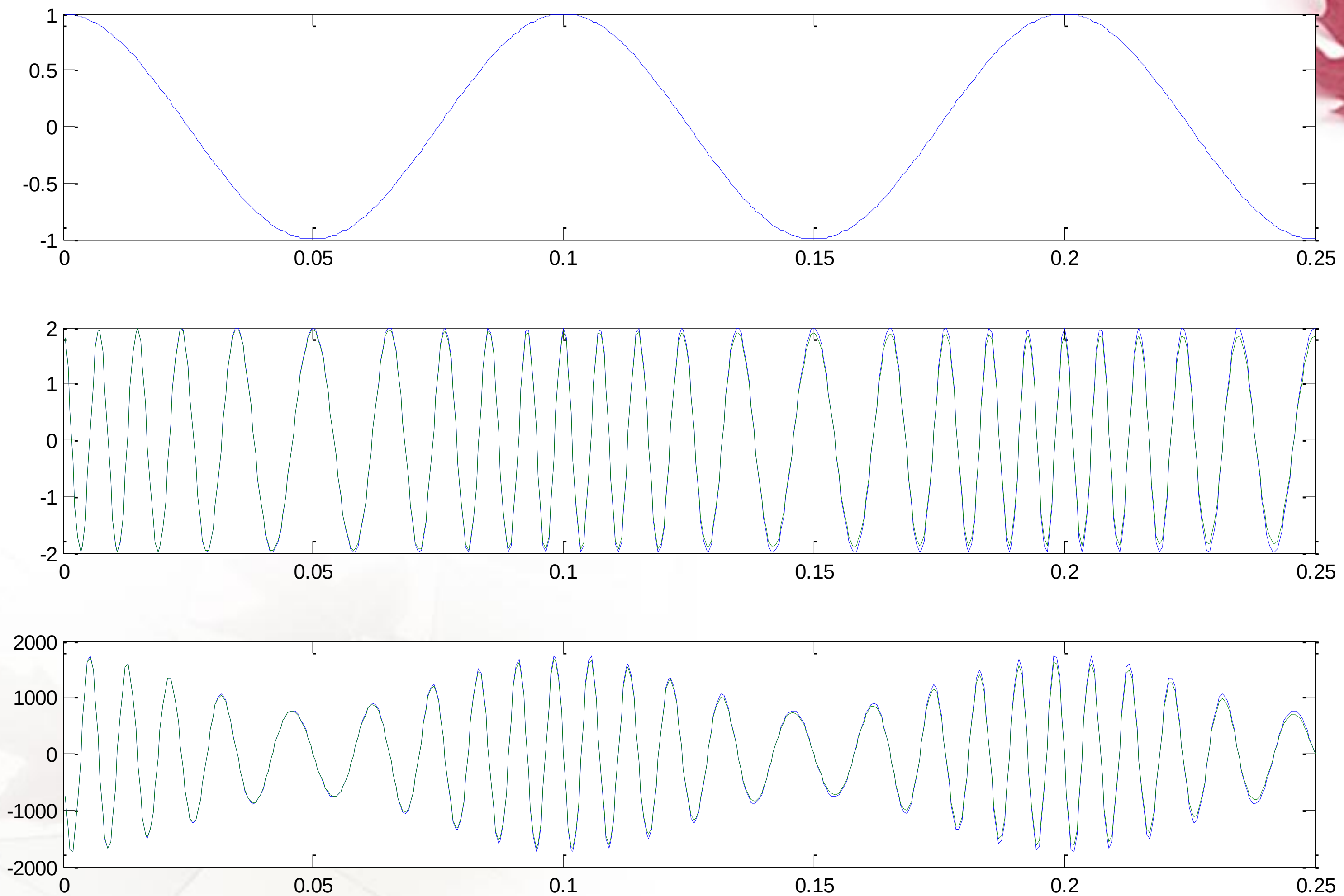
- The power of the received signal is $A^2/2$.
- The received power is inversely proportional to the square of the distance between the transmitter and the receiver.
- The power can also fluctuate due to conditions between the transmitter and receiver (rain, obstructions etc)
- This variation in received power causes fluctuations in the received amplitude.
- $r(t) = A(t)\cos(\theta_i(t))$.

Differentiator and envelope detector when received amplitude fluctuates

$$
\begin{aligned}
x(t) &= \frac{dr(t)}{dt} \\
&= \frac{d}{dt}\left(A(t)\cos(\theta_i(t))\right) \\
&= -\frac{d\theta_i(t)}{dt}A(t)\sin(\theta_i(t)) + \frac{dA(t)}{dt}\cos(\theta_i(t)) \\
&= 2\pi A(t) f_i(t)\sin\left(2\pi f_c t + 2\pi k_f \int m(t)dt + \pi\right) \\
&= 2\pi A(t)\left(f_c + k_f m(t)\right)\sin\left(2\pi f_c t + 2\pi k_f \int m(t)dt + \pi\right) \\
&\quad + \frac{dA(t)}{dt}\cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt\right)
\end{aligned}
$$

Examples

- Example 1
- $m(t) = \cos 2\pi 10t$, $f_c = 100$, $A(t) = 2e^{-t/3}$, $k_f = 40$ Hz/V.
- $s_{FM}(t) = 2\cos(2\pi 200t + 4\sin 2\pi 10t)$
- $x(t) = 4\pi e^{-t/3}(100 + 40\cos 2\pi 10t)\sin(2\pi 100t + 4\sin 2\pi 10t + \pi) - (2/3)e^{-t/3}\cos(2\pi 200t + 4\sin 2\pi 10t)$
- Example 2
- $m(t) = \cos 2\pi 10t$, $f_c = 100$, $A(t) = 2(1-t)$, $k_f = 40$ Hz/V.
- $s_{FM}(t) = 2\cos(2\pi 200t + 4\sin 2\pi 10t)$
- $x(t) = 4\pi(1-t)(100 + 40\cos 2\pi 10t)\sin(2\pi 100t + 4\sin 2\pi 10t + \pi) - 2t\cos(2\pi 200t + 4\sin 2\pi 10t)$


1
0.5
0
-0.5
-1
0
0.05
0.1
0.15
0.2
0.25
2
1
0
-1
-2
0
0.05
0.1
0.15
0.2
0.25
2000
1000
0
-1000
-2000
0
0.05
0.1
0.15
0.2
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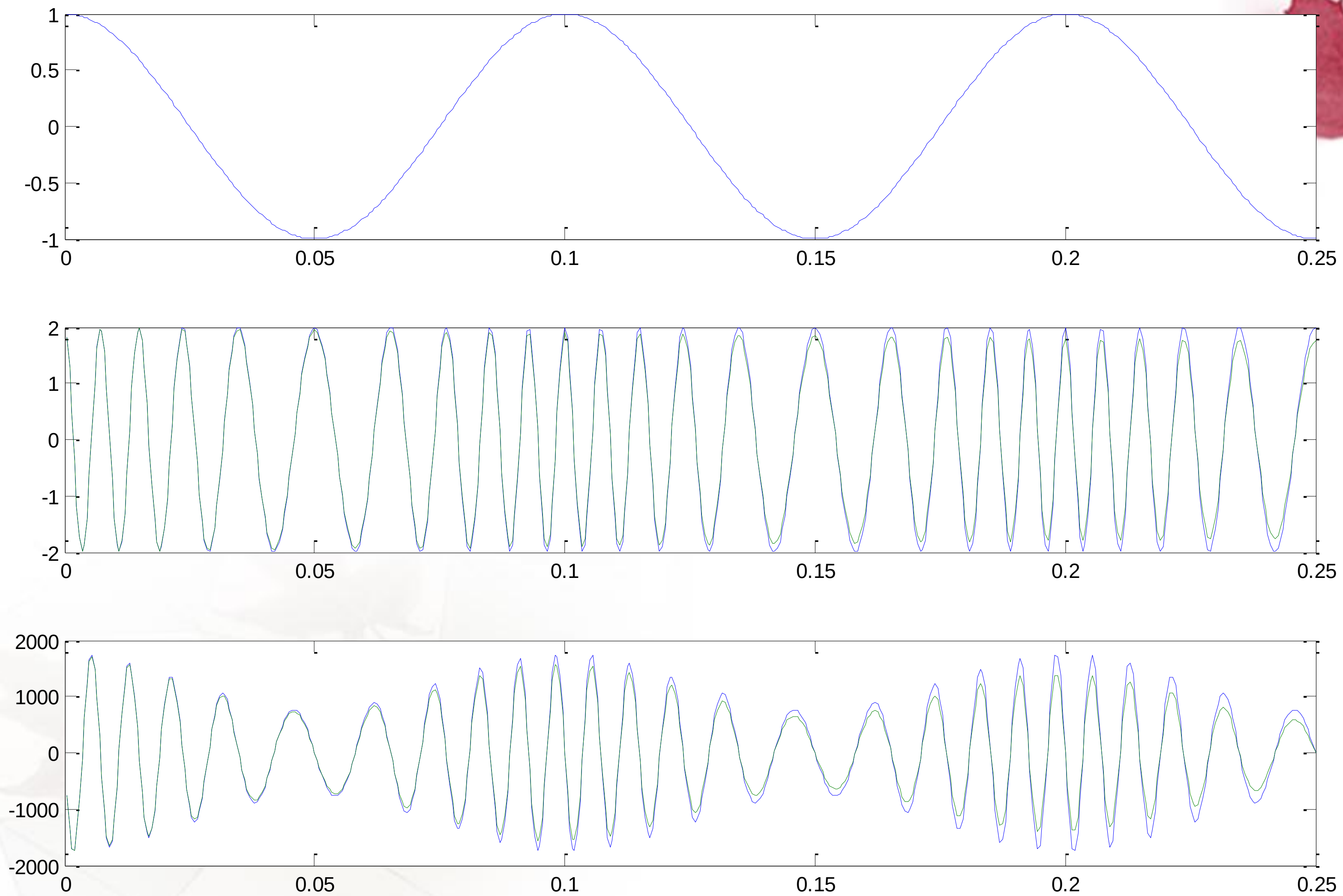
1
0.5
0
-0.5
-1
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0.05
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0.15
0.2
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2
1
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-1
-2
0
0.05
0.1
0.15
0.2
0.25
2000
1000
0
-1000
-2000
0
0.05
0.1
0.15
0.2
0.25

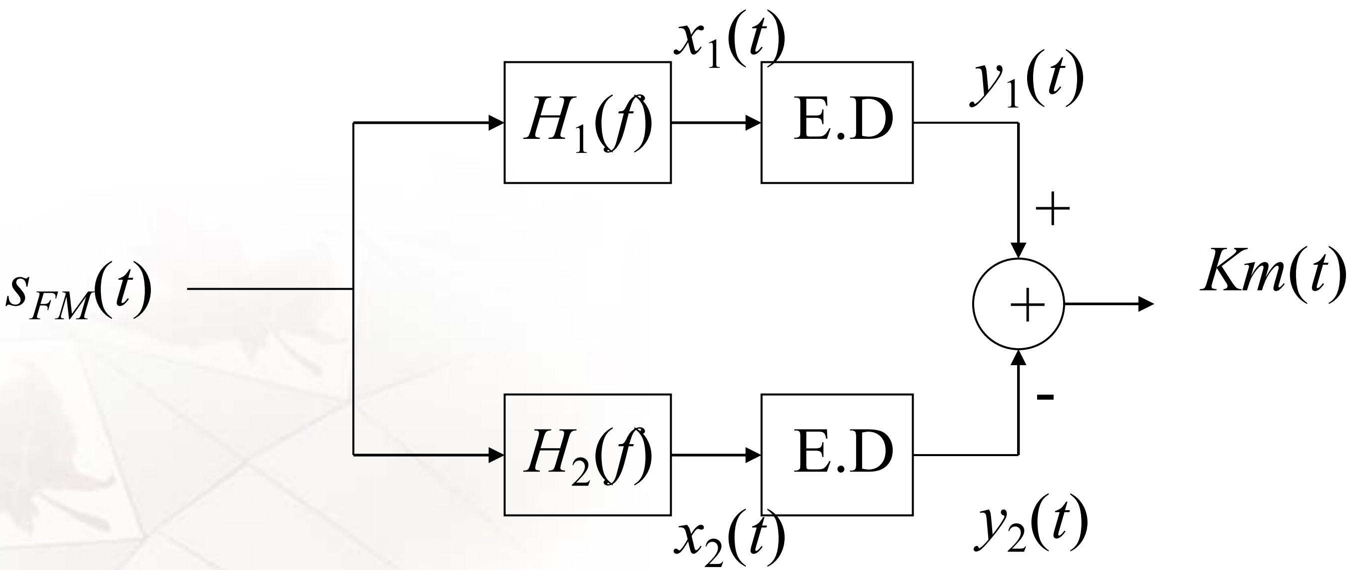
Conclusion

- When the amplitude varies the output of the envelope detector will be a distorted version of m(t).
- Solution: 1) passband limiter to make the received amplitude a constant , 2) detector that does not require taking the derivative (frequency counter of Lab 3).

- Another problem with the differentiator is that the output has a very high amplitude due to the magnitude of f_c.
  - Frequency discriminator

Frequency discriminator

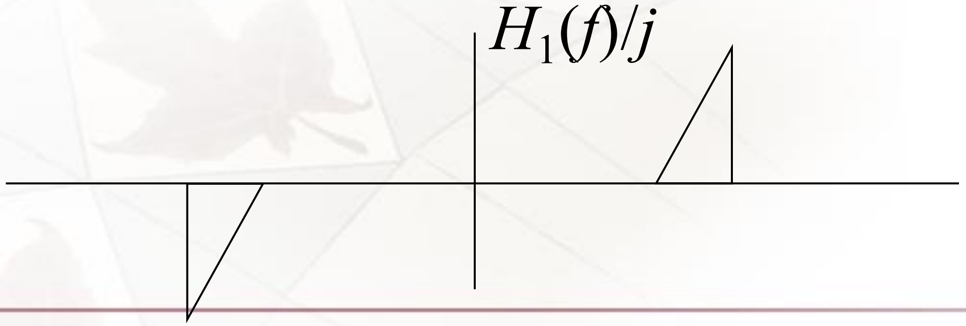
- Similar to differentiator
- Input to envelope detector has lower amplitude.

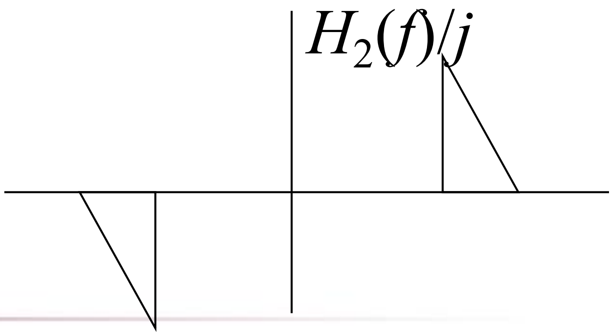


$H_1(f)$ and $H_2(f)$

$$H_1(f)=\begin{cases} j2\pi a(f-f_c+\frac{B}{2}), & f_c-\frac{B}{2}<f<f_c+\frac{B}{2} \\ j2\pi a(f+f_c-\frac{B}{2}), & -f_c-\frac{B}{2}<f<-f_c+\frac{B}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$H_2(f)=\begin{cases} -j2\pi a(f-f_c-\frac{B}{2}), & f_c-\frac{B}{2}<f<f_c+\frac{B}{2} \\ -j2\pi a(f+f_c+\frac{B}{2}), & -f_c-\frac{B}{2}<f<-f_c+\frac{B}{2} \\ 0, & \text{otherwise} \end{cases}$$





$S_{FM}(f)H_1(f)$ and its complex envelope

- $X_1(f) = S_{FM}(f)H_1(f) = (1/2)S_+(f)j2\pi a(f-f_c+B/2) + (1/2)S_-(f)j2\pi a(f+f_c-B/2)$
- The FT of the complex envelope of $x_1(t)$ is:

$$\begin{aligned}\tilde{X}_1(f) &= S_+(f+f_c)j2\pi a(f+\tfrac{B}{2}) \\ &= \tilde{S}_{FM}(f)j2\pi a(f+\tfrac{B}{2}) \\ &= aj2\pi f\tilde{S}_{FM}(f)+aj2\pi\tfrac{B}{2}\tilde{S}_{FM}(f) \\ \tilde{x}_1(t) &= a\left(\frac{d\tilde{s}_{FM}(t)}{dt}+j\pi B\tilde{s}_{FM}(t)\right)\end{aligned}$$

similarly $\tilde{x}_2(t) = a\left(\frac{d\tilde{s}_{FM}(t)}{dt}+j\pi B\tilde{s}_{FM}(t)\right)$

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$$s_+(t) = A_c e^{j\theta_i(t)} = A_c e^{j(2\pi f_c t + 2\pi k_f \int m(t)dt)}$$

$$\tilde{s}_{FM}(t) = A_c e^{j2\pi k_f \int m(t)dt}$$

$$\frac{d\tilde{s}_{FM}(t)}{dt} = j2\pi k_f A_c m(t) e^{j2\pi k_f \int m(t)dt}$$

$$\tilde{x}_1(t) = aj\pi A_c B\left[1 + \frac{2k_f m(t)}{B}\right] e^{j2\pi k_f \int m(t)dt}$$

$$\tilde{x}_2(t) = aj\pi A_c B\left[1 - \frac{2k_f m(t)}{B}\right] e^{j2\pi k_f \int m(t)dt}$$

$$x_1(t) = \pi A_c Ba\left[1 + \frac{2k_f m(t)}{B}\right] \cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt + \frac{\pi}{2}\right)$$

$$x_2(t) = \pi A_c Ba\left[1 - \frac{2k_f m(t)}{B}\right] \cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt + \frac{\pi}{2}\right)$$

$$y_1(t) - y_2(t) = 4\pi A_c a k_f m(t) = Km(t)$$