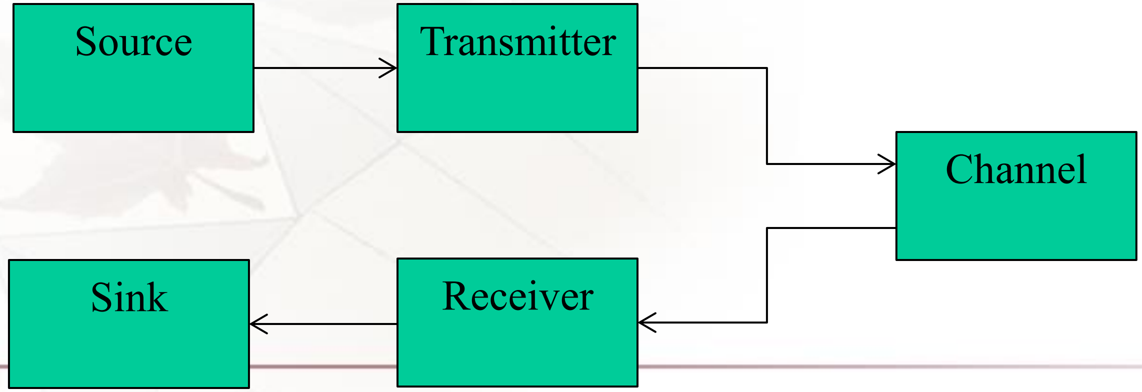
ELG3175 Introduction to Communication Systems

Introduction and Review of LTI Systems and Convolution

Introduction

- Communications in EE refers to the transmission of information from one point to another using electrical systems.
- Organization of a communication system is shown below:



Elements of a Communication System

- Source
  - Produces a message signal
- Sink
  - Intended recipient of the message signal
- Channel
  - The medium over which the information will be transmitted from the source to the sink
- Transmitter
  - Message signal may not be in a suitable form for transmission over the channel. Transmitter converts message to a suitable form

Elements of a Communication System

- Receiver
  - The receiver converts the output of the channel into a form that is suitable for use by the sink. The receiver must do this without sacrificing the content or quality of the message.

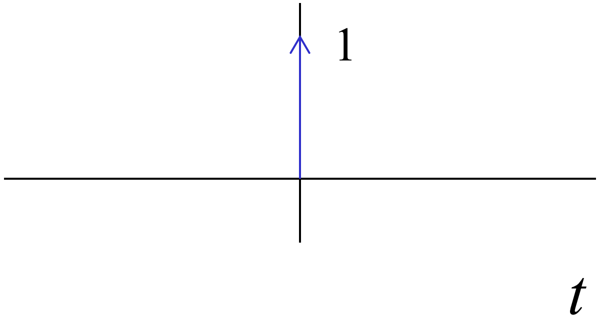
Goal of Communications Engineer

- To design transmitters and receivers that are
  - Cost efficient
  - Bandwidth efficient
  - Maximize information transfer (message at sink is a faithful representation of the source message).
  - Power efficient (uses as little power as necessary)
- Many of the above goals are contrary to one another
  - For example, one way to improve message fidelity at the receiver is to increase transmit power.
  - Therefore tradeoffs are required.

Common signals

Impulse

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

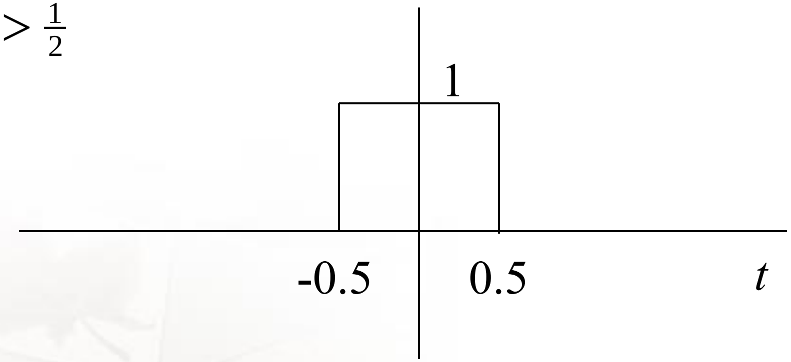


$$\int_{0^-}^{0^+} \delta(t)dt = 1$$

Common signals

- Square pulse

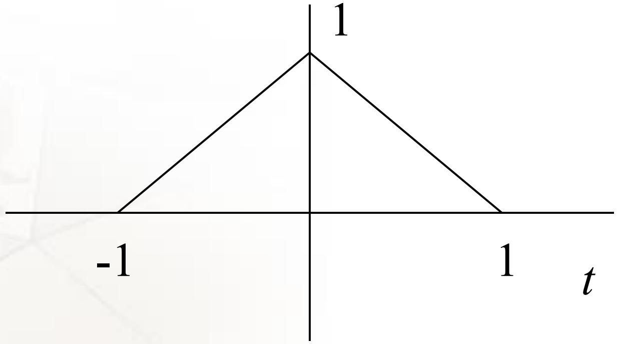
$$\Pi(t)=\begin{cases}1 & -\frac{1}{2}<t<\frac{1}{2}\\ 0 & |t|>\frac{1}{2}\end{cases}$$



Common Signals

- Triangle pulse

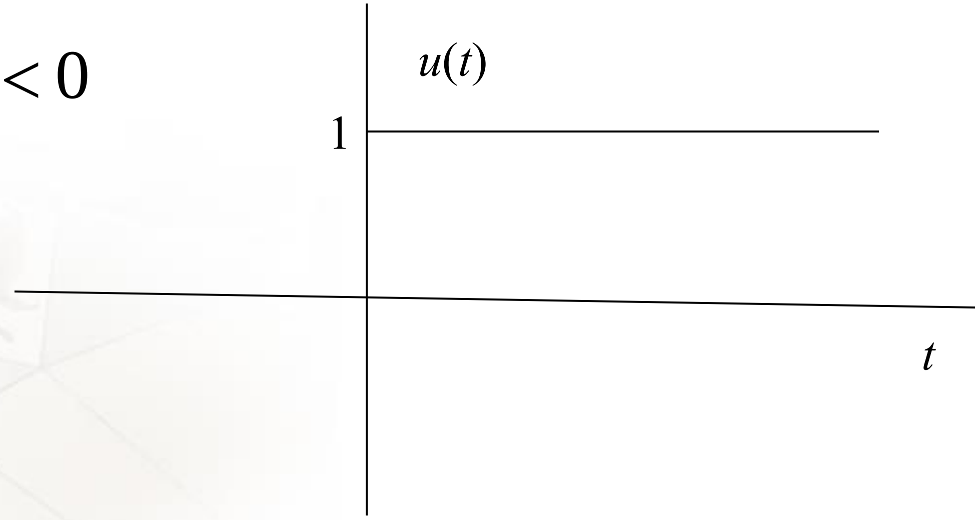
$$\Lambda(t) = \begin{cases} 1 - |t| & |t| < 1 \\ 0 & |t| > 1 \end{cases}$$



Common signals

- Step function

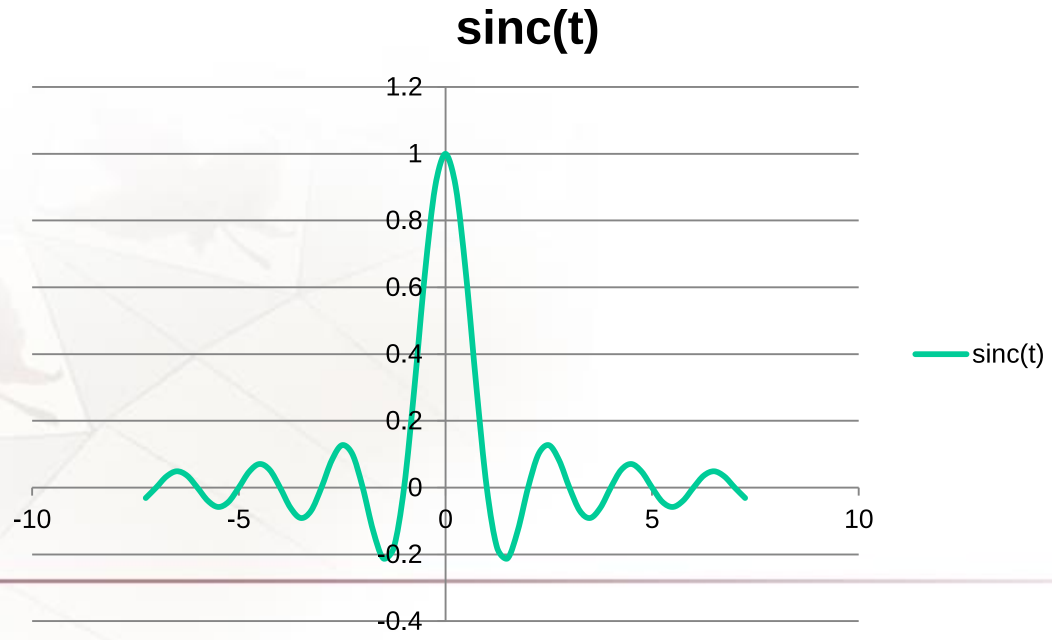
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



Common signals

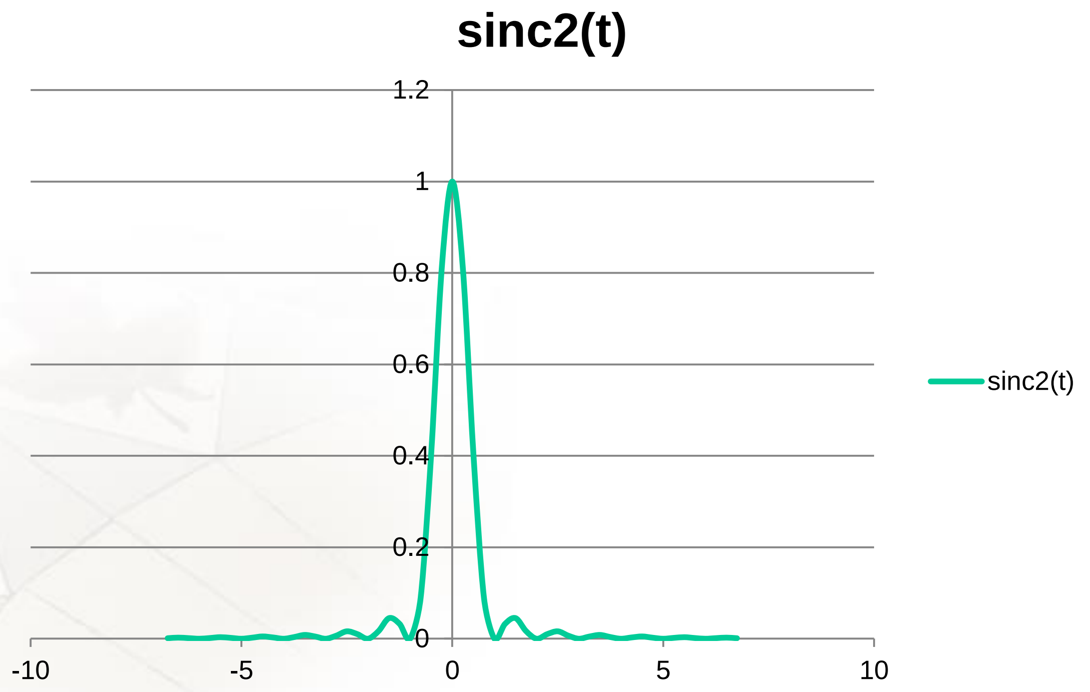
- Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



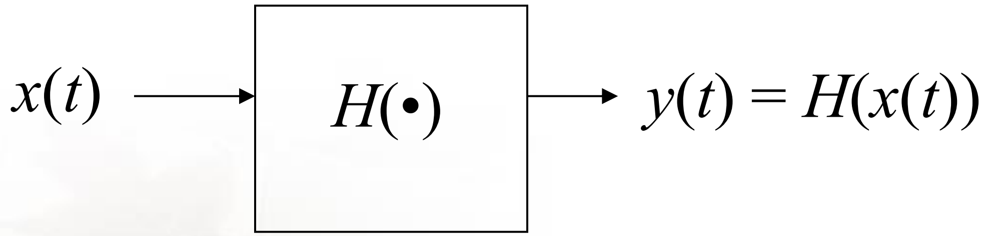
Common signals

- Sinc squared function



Review of Linear Time Invariant Systems

- A system performs a transformation on an input x(t) to produce an output y(t).



Linear Systems

- A linear system is a system for which the superposition property applies.
  - Consider a system that produces output $y_1(t)$ for input $x_1(t)$ and output $y_2(t)$ for input $x_2(t)$ then we write
    - $y_1(t) = H(x_1(t))$ and
    - $y_2(t) = H(x_2(t))$
  - Then the system H is linear if for $x_3(t) = ax_1(t)+bx_2(t)$, $y_3(t)=H(x_3(t)) = aH(x_1(t))+bH(x_2(t)) = ay_1(t)+by_2(t)$.

Example 1

- Consider the following system: $y(t) = x^2(t)$.
- For input $x_1(t)$, the output is $y_1(t) = x_1^2(t)$ and for input $x_2(t)$, the output is $y_2(t) = x_2^2(t)$.
- For input $x_3(t) = \alpha x_1(t) + \beta x_2(t)$, the output is $y_3(t) = x_3^2(t) = (\alpha x_1(t) + \beta x_2(t))^2 = \alpha^2 x_1^2(t) + 2\alpha\beta x_1(t) x_2(t) + \beta^2 x_2^2(t)$.
- If the system is linear, $y_3(t)$ should be $\alpha y_1(t) + \beta y_2(t) = \alpha x_1^2(t) + \beta x_2^2(t) \neq y_3(t)$; therefore the system is not linear.

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Example 2

- A system has input-output relationship $y(t) = tx(t)$.
- For input $x_3(t) = \alpha x_1(t) + \beta x_2(t)$, the output is $y_3(t) = t(\alpha x_1(t) + \beta x_2(t)) = \alpha(tx_1(t)) + \beta(tx_2(t)) = \alpha y_1(t) + \beta y_2(t)$.
- Therefore the system is linear.

Time Invariant Systems

- A system is time invariant if a time shift to the input results in no changes other than the same time shift being applied to the output.
- If $y_1(t)$ is the output of the system when $x_1(t)$ is the input let $x_2(t) = x_1(t-\tau)$ be the input that produces output $y_2(t)$.
- The system is time invariant if $y_2(t) = y_1(t-\tau)$.

Example

- Consider again the system whose input-output relationship is $y(t) = tx(t)$, therefore $y_1(t) = tx_1(t)$.
- Let $x_2(t) = x_1(t-\tau)$. The corresponding output is $y_2(t) = tx_2(t) = tx_1(t-\tau)$.
- However, $y_1(t-\tau) = (t-\tau)x_1(t-\tau)$, therefore this system is not time invariant.

Example 2

- Consider the following system: $y(t) = 3+4x^2(t)$, therefore $y_1(t) = 3+4x_1^2(t)$.
- Let$x_2(t) = x_1(t-\tau)$, therefore $y_2(t) = 3+4x_2^2(t) = 3+4x_1^2(t-\tau) = y_1(t-\tau)$.
- This system is time invariant.

Linear time invariant systems

- A system is LTI if it is both linear and time invariant.
- An LTI system is described by its impulse response.
- The system's impulse response is $h(t)$ and it is the output of the system when the input is $x(t) = \delta(t)$.
- Properties of $\delta(t)$.
  - $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$.
  - $\int_{-\infty}^{\infty} \delta(t)dt = 1$.
  - $\int_{-\infty}^{\infty} x(t)\delta(t-\tau)dt = x(\tau)$.

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Output of LTI system

- For any input $x(t)$, the output of an LTI system is $y(t) = x(t)*h(t)$, where * denotes convolution.

$$x(t)*h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

Properties

- We can show that $x(t)*(y_1(t) + y_2(t)) = x(t)*y_1(t) + x(t)*y_2(t)$. Let $z(t) = y_1(t) + y_2(t)$.

$$
\begin{aligned}
x(t)*(y_1(t)+y_2(t)) = x(t)*z(t) \\
&= \int_{-\infty}^{\infty} x(\lambda)z(t-\lambda)d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda)\left[y_1(t-\lambda)+y_2(t-\lambda)\right]d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda)y_1(t-\lambda)d\lambda + \int_{-\infty}^{\infty} x(\lambda)y_2(t-\lambda)d\lambda \\
&= x(t)*y_1(t)+x(t)*y_2(t)
\end{aligned}
$$

Convolution with impulse function

$$\begin{aligned} x(t) * \delta(t) &= \int_{-\infty}^{\infty} x(\lambda)\delta(t-\lambda)d\lambda \\ &= \int_{-\infty}^{\infty} \delta(\lambda)x(t-\lambda)d\lambda \\ &= x(t) \end{aligned}$$

$$\begin{aligned} x(t) * \delta(t-\tau) &= \int_{-\infty}^{\infty} x(\lambda)\delta(t-\tau-\lambda)d\lambda \\ &= \int_{-\infty}^{\infty} \delta(\lambda-\tau)x(t-\lambda)d\lambda \\ &= x(t-\tau) \end{aligned}$$

Example

- $y(t) = \Pi(t) * \Pi(t)$
  - Use drawings to help find the limits of integration.
  - Function mappings to map $\Pi(t)$ onto $\Pi(t-\lambda)$ on λ scale.

Causality

- A system is causal if it's output depends only on past and present values of the input (it does not depend on future values of the input).
- For LTI system:

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

- When $\lambda < 0$, $y(t)$ depends on future values of the input. Therefore an LTI system is causal if $h(\lambda)=0$ for all $\lambda < 0$.

Stability

- A system is stable if for any bounded input, it's output is also bounded.
- For LTI system, this implies that

$$\int_{-\infty}^{\infty} |h(\lambda)|\, d\lambda \leq \infty$$