(A1) Closure under addition: If \( a \) and \( b \) belong to \( S \), then \( a + b \) is also in \( S \)

(A2) Associativity of addition: \( a + (b + c) = (a + b) + c \) for all \( a, b, c \) in \( S \)

(A3) Additive identity: There is an element 0 in \( R \) such that \( a + 0 = 0 + a = a \) for all \( a \) in \( S \)

(A4) Additive inverse: For each \( a \) in \( S \) there is an element \(-a\) in \( S \) such that \( a + (-a) = (-a) + a = 0 \)

(A5) Commutativity of addition: \( a + b = b + a \) for all \( a, b \) in \( S \)

(M1) Closure under multiplication: If \( a \) and \( b \) belong to \( S \), then \( ab \) is also in \( S \)

(M2) Associativity of multiplication: \( a(bc) = (ab)c \) for all \( a, b, c \) in \( S \)

(M3) Distributive laws: \( a(b + c) = ab + ac \) for all \( a, b, c \) in \( S \)

(M4) Commutativity of multiplication: \( ab = ba \) for all \( a, b \) in \( S \)

(M5) Multiplicative identity: There is an element 1 in \( S \) such that \( 1a = a1 = a \) for all \( a \) in \( S \)

(M6) No zero divisors: If \( a, b \) in \( S \) and \( ab = 0 \), then either \( a = 0 \) or \( b = 0 \)

(M7) Multiplicative inverse: If \( a \) belongs to \( S \) and \( a \neq 0 \), there is an element \( a^{-1} \) in \( S \) such that \( a a^{-1} = a^{-1}a = 1 \)

Figure 4.2  Group, Ring, and Field