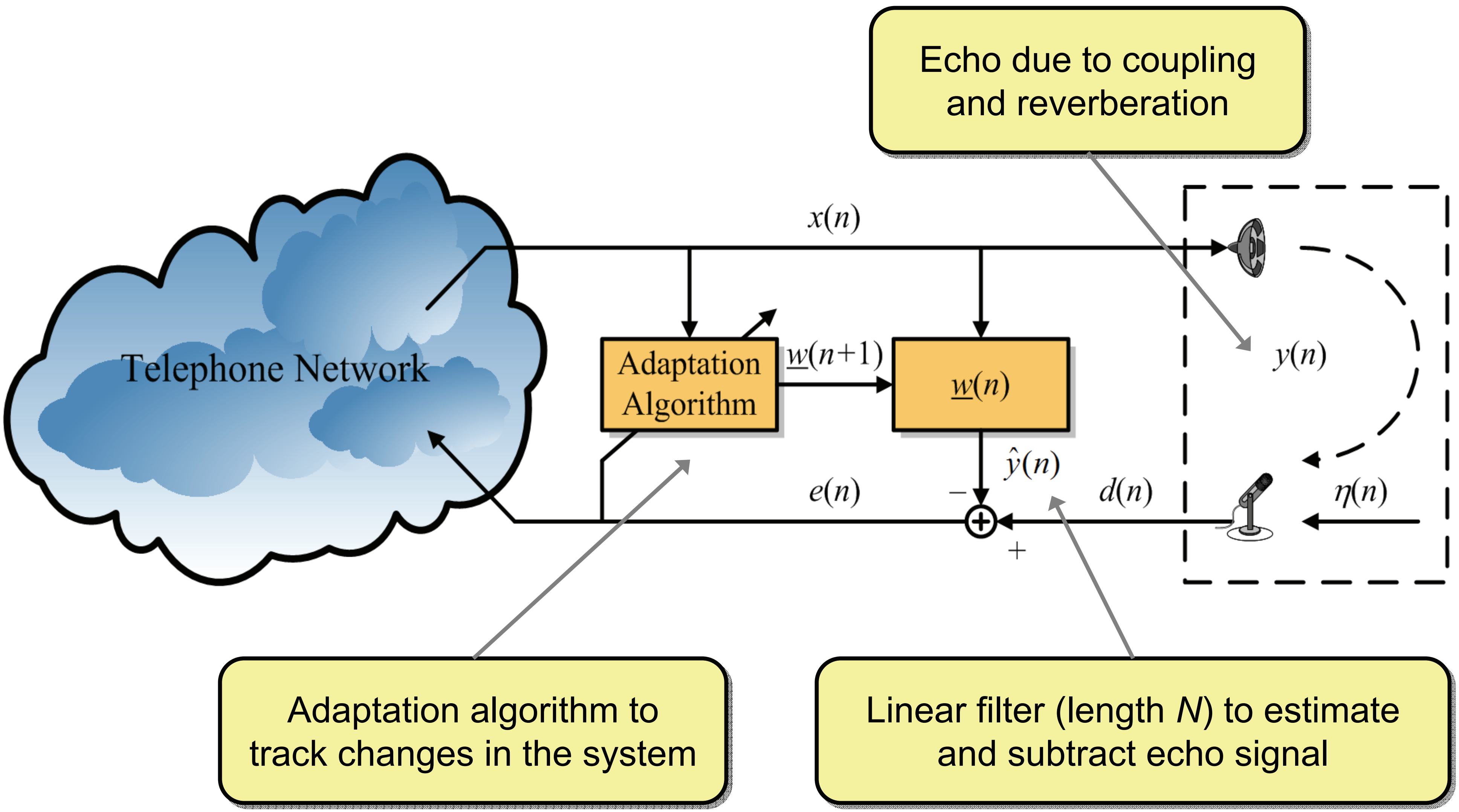
Abstract

This paper proposes a selective coefficient update algorithm for reducing the complexity of the proportionate normalized least-mean-square (P-NLMS) class of algorithms. It is shown that an optimal subset of coefficients to update, namely those minimizing the *a posteriori* error, cannot be constructed efficiently. A sub-optimal block-based coefficient selection algorithm is presented that combines proportional weighting of the input signal vector with fast ranking methods. It is compared to existing sub-optimal algorithms with respect to complexity overhead and convergence rate. Simulations show that the proposed algorithm produces performance approaching that of the optimal subset while maintaining a low coefficient selection overhead.

Echo Canceller Structure (Acoustic)



Adaptation Algorithms

- NLMS variants are commonly employed in practice to update filter coefficient vector:

$$e(n) = d(n) - \hat{y}(n) = d(n) - \underline{x}^T(n)\underline{w}(n)$$

$$w_j(n+1) = w_j(n) + \frac{\mu x(n-j)e(n)}{\underline{x}^T(n)\underline{x}(n) + \delta_{NLMS}}$$

- Complexity ~2*N* operations per sample period

Proportionate NLMS

- Basic idea:
  - Employ per-coefficient step sizes proportional to filter coefficient magnitude
  - Results in a family of related algorithms: P-NLMS, IP-NLMS, GP-NLMS

$$w_j(n+1) = w_j(n) + \frac{\mu g_j(n) x(n-j) e(n)}{\underline{x}^T(n)\underline{G}(n)\underline{x}(n) + \delta_{IPNLMS}}$$

- Faster convergence rate, but complexity increases to ~4*N* operations per sample

Selective Coefficient Update

- Basic idea:
  - Update only a subset of $M < N$ coefficients per sample period
  - Save up to $N - M$ operations, *provided* coefficient selection requires fewer operations

Can selective coefficient update reduce the complexity of proportionate NLMS algorithms?

Optimal Coefficient Selection

- Goal: minimize the *a posteriori* error:

$$\varepsilon(n) = d(n) - \underline{x}^T(n)\underline{w}(n+1)]$$

$$\varepsilon^2(n) = e^2(n)\left[1 - \frac{\mu\sum_{j=0}^{N-1} g_j(n)x^2(n-j)}{\underline{x}^T(n)\underline{G}(n)\underline{x}(n) + \delta_{IPNLMS}}\right]$$

- Select *M* coefficients with largest weighted update term
- Requires a full sort involving O($N\log_2 N$) operations → impractical

Sub-Optimal Coefficient Selection

- Divide filter coefficient and input signal vectors into *B* = *N* / *L* equal-sized blocks:

$$\underline{x}(n) = \underline{x}_1(n) \,|\, \cdots \,|\, \underline{x}_B(n) \qquad \underline{w}(n) = \underline{w}_1(n) \,|\, \cdots \,|\, \underline{w}_B(n)$$

- BSC-IP-NLMS (Tanrikulu & Dogancay):
  - Calculate block power and sort:

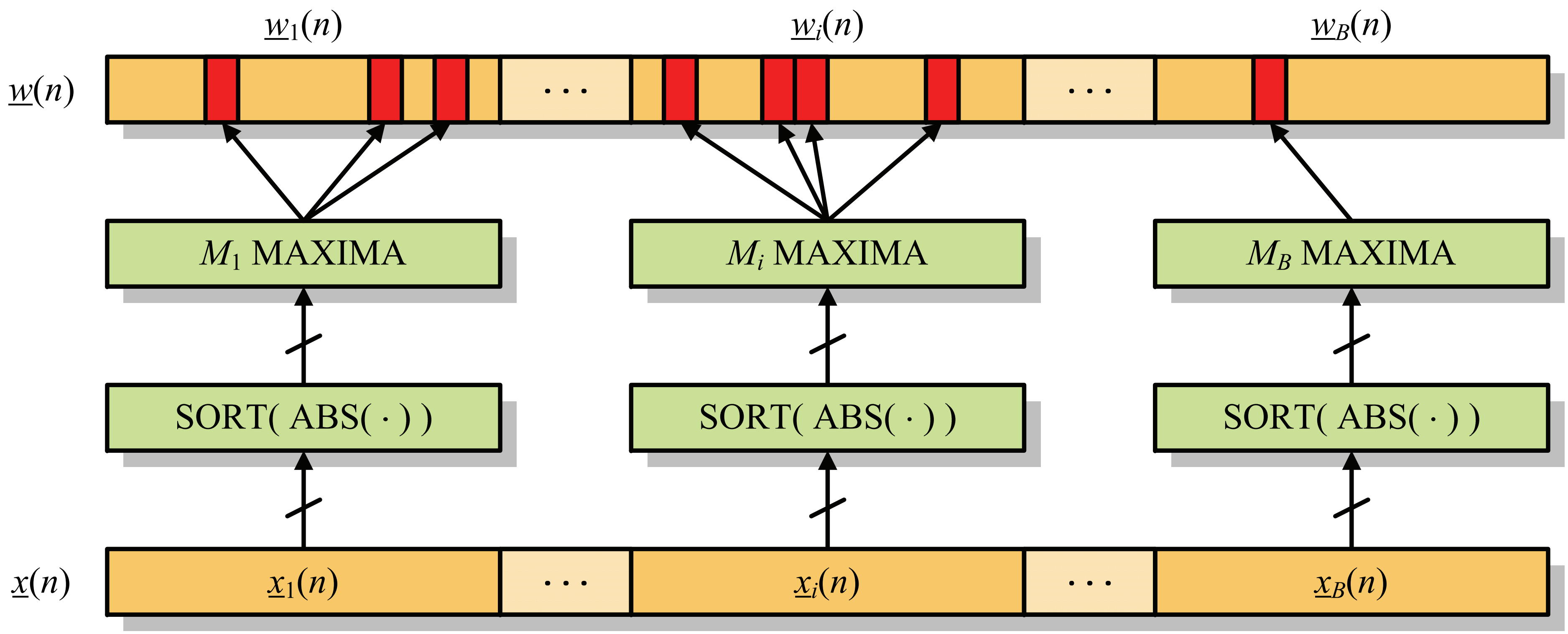
$$sort\{\underline{x}_1^T(n)\underline{G}_1(n)\underline{x}_1(n), \cdots, \underline{x}_B^T(n)\underline{G}_B(n)\underline{x}_B(n)\}$$

  - Update coefficient *blocks* containing maximal power
  - $O(B\log_2 B)$ operations per sample period, optimal as $B \rightarrow N$

Proposed Approach

- Variable selective coefficient (VSC-IP-NLMS)
- Key ideas:
  - For each block *i*, update M_i coefficients with largest corresponding input sample magnitudes
  - Periodically re-allocate updates per block based on filter coefficient variability → M_i variable
  - Total number of updates: $M_1 + M_2 + \cdots + M_B = M$

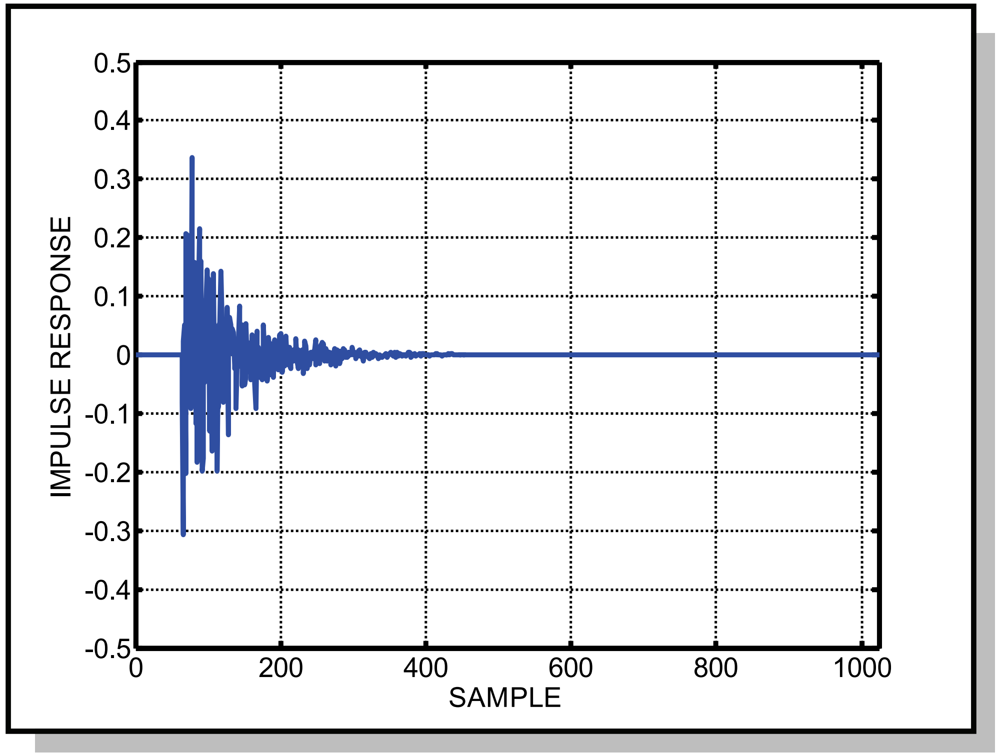
Proposed Approach (cont.)



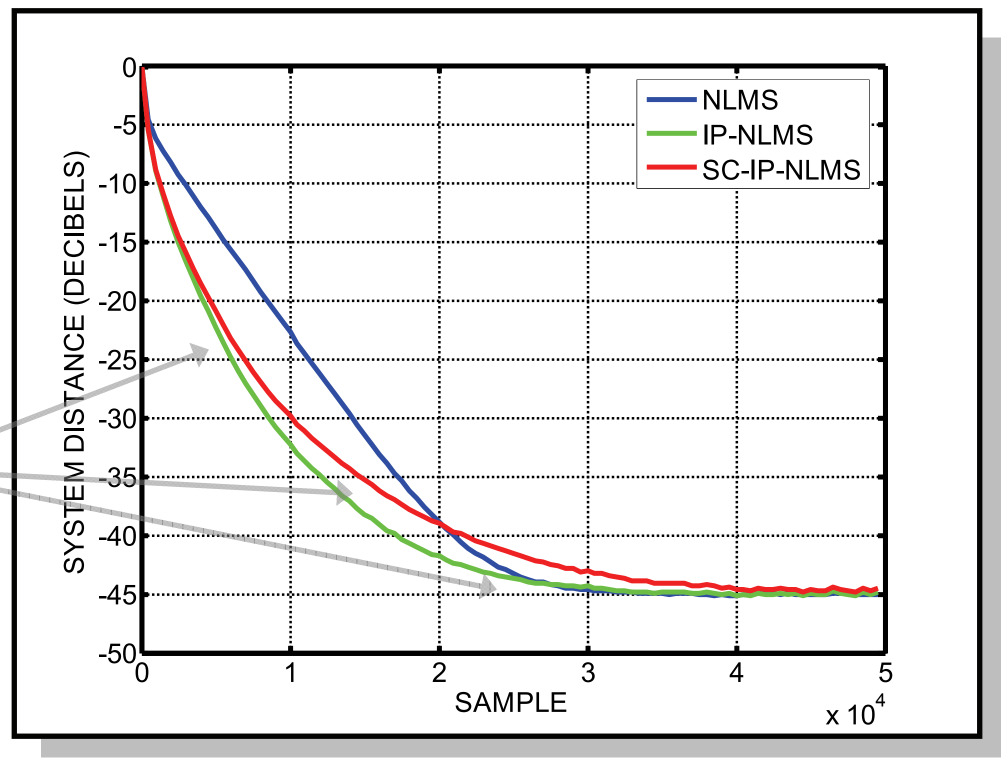
- Using input signal magnitude guarantees coefficient updates in every block
- Requires B sorted input signal lists (one per block), but list update can be done in ~$B(2\log_2 L+2)$ operations

Simulation Setup

- Acoustic echo path (*N* = 1024)
- AR(2) input signal
- Selective coefficient update applied to IP-NLMS
- Coefficient selection methods (*M* = 384 out of 1024)
  - Optimal (SC-IP-NLMS)
  - Block update (BSC-IP-NLMS)
  - Proposed (VSC-IP-NLMS)

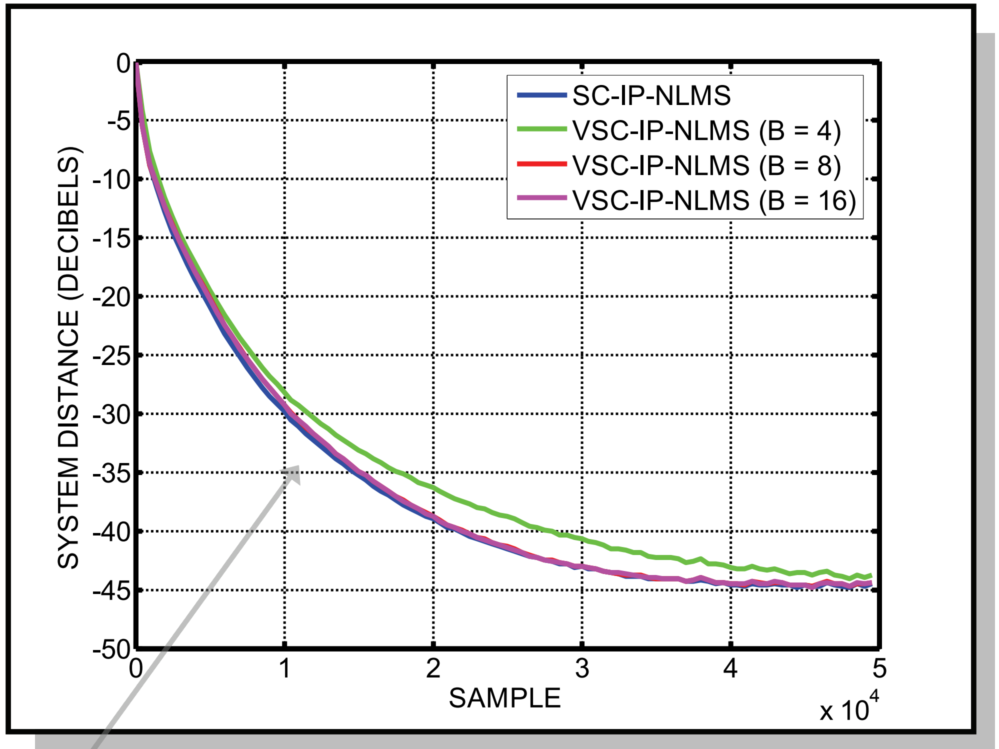


Baseline comparison:
•IP-NLMS improves upon NLMS
•SC-IP-NLMS exhibits some degradation (expected)
•Similar steady-state error

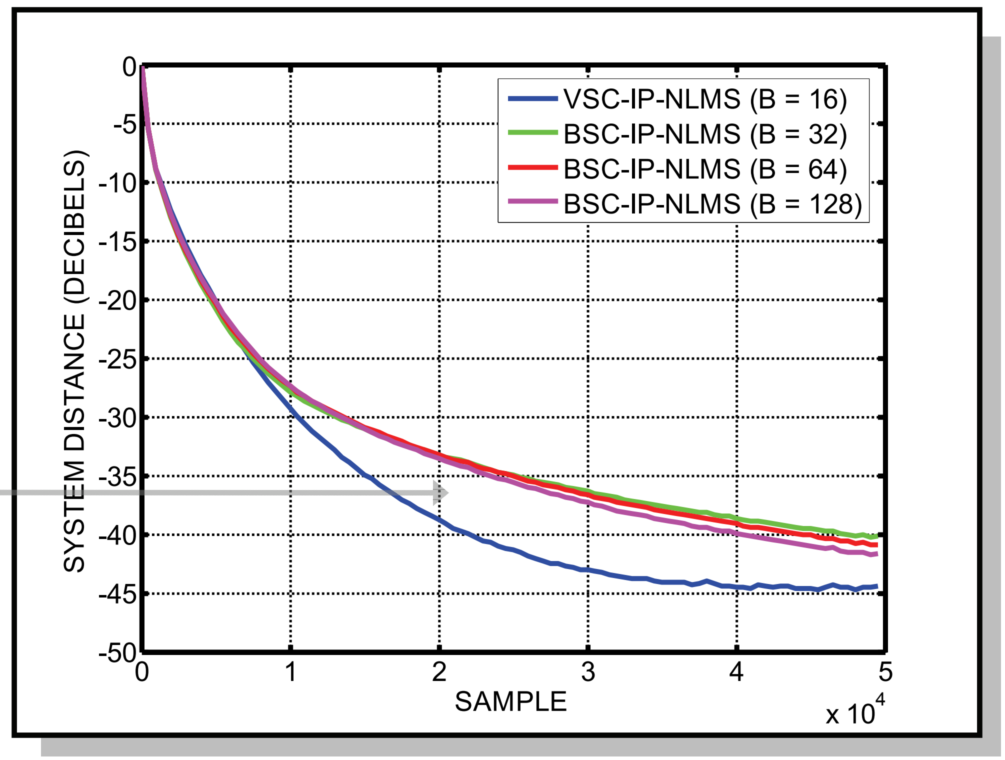


Comparison of Coefficient Selection Methods

- SC-IP-NLMS compared to VSC-IP-NLMS and BSC-IP-NLMS for various block sizes



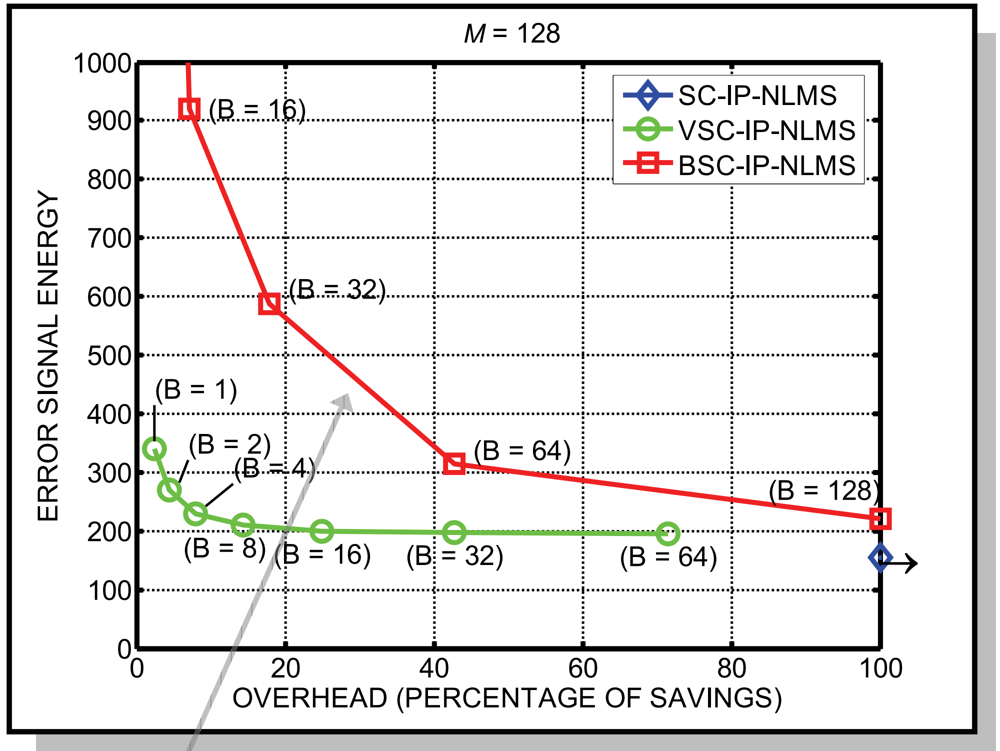
VSC-IP-NLMS approaches optimal coefficient selection as number of blocks increases



Block-only update appears to slow down → only maximum-magnitude blocks are updated

Performance Versus Coefficient Selection Overhead

- Error signal energy calculated over initial convergence period
- Coefficient selection overhead:
  - VSC-IP-NLMS: ~$B(2\log_2(L)+2)$
  - BSC-IP-NLMS: ~$B\log_2 B$



Observations:
- BSC-IP-NLMS: generally degraded performance as *M* decreases
- VSC-IP-NLMS: less variability as block sizes change → more choice in allowable overhead