Active Noise Control (ANC): definition and applications

- definition: use of electroacoustic or electro-mechanical transducers to cancel a noise source, based on the physical principle of wave superposition.
- some applications:
  - attenuation of noise from ducts or pipework in heating, ventilation, and air conditioning (HVAC)
  - creation of "quiet zones" in an enclosed room, a vehicle enclosure, or an aircraft cabin
  - attenuation of engine exhaust noise and axial flow fans radiated noise
  - active headsets, combined audio/ANC systems or communications/ANC systems
  - reduction of free-field radiation, such as for electrical transformer noise
  - active vibration control: modal attenuation of vibrations on a beam or a plate

Classification of ANC systems

- feedforward: a coherent reference noise input is sensed before it propagates past the control transducer, and feedback: no reference input used
- broadband: wideband e.g. using a microphone reference, and narrowband: mostly tonal control e.g. using a tachometer reference
- single-channel systems and multiple-channel systems (multiple transducers, error sensors, and/or reference sensors)
- mostly digital controllers, based on adaptive filtering (FIR, IIR) or control theory, or a mixture
- this current work:
  - mostly addresses broadband multichannel feedback FIR adaptive filter-based ANC
  - also applicable to single channel, narrowband, or feedback systems

FIR adaptive filtering for ANC

- several systems are based on stochastic gradient steepest descent algorithms, such as filtered-x LMS
- some architectures (e.g. delay-compensated structures) are suitable for faster converging algorithms, such as recursive least-squares algorithms
- Fast affine projection (FAP) algorithms for ANC:
  - introduced for single channel in 1995 and for multi channel cases in 2003
  - provide a good performance – complexity tradeoff
  - for a filtered-x based structure with noisy models, can outperform more complex least-squares based algorithms
  - complexity is often of the same order as basic LMS-based schemes

Why yet another FAP algorithm for ANC?

- a few more alternative FAP or pseudo-FAP algorithms for ANC have been introduced since 2003
- they are either non-exact FAP-like algorithms, or non-relaxed FAP-like algorithms
- practical implementation of these algorithms has shown a great sensitivity to parameter tuning (step size, regularization factor)
- there is a need for a simple, efficient implementation, not too sensitive to parameter tuning, i.e. more robust

Resulting MFX-LDL^T-FAP algorithm

\[ y_f(n) = r_f(n-1) + \sum_{j=1}^{N} \alpha_j(n) y_f(n-1) + \sum_{j=1}^{N} \alpha_j(n) r_f(n-1) \]  
\[ \alpha_j(n) = \beta_j(n) - \sum_{i=1}^{J} \hat{J}_j \hat{Y}_i(n) \]  
\[ \hat{J}_j(n) = \sum_{i=1}^{N} \beta_i(n) \hat{Y}_i(n) \]  
\[ \hat{Y}_i(n) = \sum_{j=1}^{J} \beta_j(n) Y_i(n) \]  
\[ \beta_j(n) = \sum_{i=1}^{N} \beta_i(n) Y_i(n) \]  
\[ \beta_i(n) = \sum_{j=1}^{J} \beta_j(n) Y_j(n) \]  
\[ \delta_j(n) = \alpha_j(n) - \sum_{i=1}^{J} \hat{J}_j \hat{Y}_i(n) \]  
\[ r(n) = r(n-1) + \sum_{j=1}^{N} \alpha_j(n-1) r_f(n-1) - \sum_{j=1}^{N} \alpha_j(n-1) r_f(n-1) \]  
\[ p(n) = p(n-1) + \sum_{j=1}^{N} \alpha_j(n-1) p_f(n-1) - \sum_{j=1}^{N} \alpha_j(n-1) p_f(n-1) \]  
\[ E(n) = E(n-1) + \sum_{j=1}^{N} \alpha_j(n-1) E_f(n-1) - \sum_{j=1}^{N} \alpha_j(n-1) E_f(n-1) \]  
\[ R(n) = R(n-1) + \sum_{j=1}^{N} \alpha_j(n-1) R_f(n-1) - \sum_{j=1}^{N} \alpha_j(n-1) R_f(n-1) \]  
\[ \hat{e}_f(n) = e_f(n-1) + \sum_{j=1}^{N} \alpha_j(n-1) \hat{e}_f(n-1) - \sum_{j=1}^{N} \alpha_j(n-1) \hat{e}_f(n-1) \]  
\[ \hat{e}_f(n) = e_f(n-1) + \sum_{j=1}^{N} \alpha_j(n-1) \hat{e}_f(n-1) - \sum_{j=1}^{N} \alpha_j(n-1) \hat{e}_f(n-1) \]

- The use of equation (8) with with \( \mu \) makes the resulting FAP algorithm relaxed, and the solving of the linear set of equations in (9) without any approximation makes the resulting FAP algorithm exact.
- To solve (9) without any approximation, the LDL^T method combined with forward and backward substitution is used, where \( D \) corresponds to a diagonal matrix, and \( L \) corresponds to a lower triangular matrix with unit values in the main diagonal.
- To reduce complexity, the LDL^T decomposition can also be performed at a reduced rate with update period \( p \).

Comparison of convergence performance and complexity

- the new proposed MFX-LDL^T-FAP algorithm, while being more complex than the MFX-GS-FAP algorithm or other recently proposed FAP-like algorithms for ANC, still has a reasonable complexity i.e. of the same order as a steepest descent LMS ANC algorithm (especially for \( p=20 \))
- the numerical robustness and good convergence properties for a wide range of step size or regularization values in the new proposed MFX-LDL^T-FAP algorithm can justify the (reasonable) extra complexity cost.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity for</th>
<th>J = 1, J = 1, K = 1</th>
<th>Complexity for</th>
<th>J = 1, J = 3, K = 2</th>
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<tr>
<td>MFX-GSFAP</td>
<td>( 858 \times 10 )</td>
<td>( 5644 \times 20 )</td>
<td>MFX-GSFAP</td>
<td>( 510 \times 0.05 )</td>
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<td>( 3440 \times 1 )</td>
<td>MFX-GSFAP</td>
<td>( 428 \times 0.1 )</td>
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<tr>
<td>MFX-GSFAP, ( N=10 ), ( p=1 )</td>
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<td>( 376 \times 2 )</td>
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<td>428 / 0</td>
<td>2208 / 0</td>
<td>MFX-GSFAP</td>
<td>510 / 0.05</td>
</tr>
</tbody>
</table>

Convergence of the affine projection algorithms.

Convergence curves for the MFX-LDL^T-FAP with different step size gain values, for \( d = \mu = 2 \).