LTL (Linear Temporal Logic)

Definition 3.5. A *path* in a model $\mathcal{M} = (S, \rightarrow, L)$ is an infinite sequence of states $s_1, s_2, s_3, \ldots$ in S such that, for each $i \geq 1$, $s_i \rightarrow s_{i+1}$. We write the path as $s_1 \rightarrow s_2 \rightarrow \ldots$

Notation. Let $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ We write π^i for the suffix starting at s_i, e.g., π^3 is $s_3 \rightarrow s_4 \rightarrow \ldots$

Definition 3.6. Let $\mathcal{M} = (S, \rightarrow, L)$ be of model and $\pi = s_1 \rightarrow \ldots$ be a path in $\mathcal{M}$. Whether π satisifes an LTL formula is defined by the satisfaction relation $\models$ as follows:

1. $\pi \models \top$
2. $\pi \not\models \bot$
3. $\pi \models p$ iff $p \in L(s_1)$.
4. $\pi \models \neg\phi$ iff $\pi \not\models \phi$.
5. $\pi \models \phi \wedge \psi$ iff $\pi \models \phi$ and $\pi \models \psi$.
6. $\pi \models \phi \vee \psi$ iff $\pi \models \phi$ or $\pi \models \psi$.
7. $\pi \models \phi \rightarrow \psi$ iff $\pi \models \psi$ whenever $\pi \models \phi$.

LTL (continued)

8. $\pi \models X\phi$ iff $\pi^2 \models \phi$.
9. $\pi \models G\phi$ iff for all $i \geq 1$, we have $\pi^i \models \phi$.
10. $\pi \models F\phi$ iff for some $i \geq 1$, we have $\pi^i \models \phi$.
11. $\pi \models \phi\ U\ \psi$ iff there exists an $i \geq 1$ such that $\pi^i \models \psi$ and for each j such that $1 \leq j < i$, we have $\pi^j \models \phi$.
12. $\pi \models \phi\ W\ \psi$ iff either there exists an $i \geq 1$ such that $\pi^i \models \psi$ and for each j such that $1 \leq j < i$, we have $\pi^j \models \phi$; or for all $k \geq 1$ we have $\pi^k \models \phi$.
13. $\pi \models \phi\ R\ \psi$ iff either there exists an $i \geq 1$ such that $\pi^i \models \phi$ and for each j such that $1 \leq j < i$, we have $\pi^j \models \psi$; or for all $k \geq 1$ we have $\pi^k \models \psi$.

Definition 3.8. Let $\mathcal{M} = (S, \rightarrow, L)$. A formula ϕ is satisfied in a state s (written $\mathcal{M}, s \models \phi$) iff for every path π starting at s, we have $\pi \models \phi$. If s is omitted (i.e., $\mathcal{M} \models \phi$), then s is understood to be the start state.