

# Petri Nets Based Formalization of Textual Use Cases

Stéphane S. Somé

School of Information Technology and Engineering (SITE) University of Ottawa  
800 King Edward, P.O. Box 450, Stn. A Ottawa, Ontario, K1N 6N5, Canada  
ssome@site.uottawa.ca

## Abstract

A Use Case is a specification of interactions involving a system and external actors of that system. The intuitive, user centered nature of textual use cases is one of the reasons for the success of the use case approach. A certain level of formalization is however needed to automate use case based system development, including tasks such as model synthesis, verification and validation. In this paper, a formalization of textual use cases is proposed. At the syntactic-level, an UML metamodel and a restricted-form of natural language are defined for use case description. Use cases execution semantics are proposed as a set of Mapping Rules from well-formed use cases to Basic Petri nets. The semantics consider use cases sequencing constraints defined at the syntactic-level. The proposed formalization serves as a basis for state-model synthesis from use cases. UML activity diagrams are generated to capture use cases sequencing and UML StateCharts to capture event flows within use cases.

**Keywords:** Use Cases, Petri nets, StateCharts, Activity diagrams, Synthesis.

## 1 Introduction

A use case is defined in the Unified Modeling Language (UML) specification as "the specification of a sequence of actions, including variants that a system (or a subsystem) can perform, interacting with actors of the system" [21]. Since their introduction by Jacobson [11], use cases are used to drive the development process from the early stages of business modeling to acceptance testing [10]. The UML defines use cases as abstract specification of behaviors. The concrete behavior corresponding to a use case is specified using various behavior description approaches such as interactions, activities, state machines, pre/post-conditions or natural language text.

For practical reasons, and in order to allow for an easy communication with stakeholders, natural language text is usually used for use cases at the early stages of development. While behavior modeling techniques such as interactions, activities and state machines have been subject to intensive formalization efforts, informal representations are essentially used for

use cases in textual form. Various *templates* and *guidelines* [2, 3] can be considered as providing some degree of formalization to textual use cases. However, the emphasis is mainly on the structure of use case documents in term of sections, and on directions on the form of natural language. Very little work has been done on the formal definition of textual use cases execution semantics. There are several potential benefits to a more formal definition of textual use cases. As requirements artifacts, it is important to ensure use cases effectively express properties that are agreed up on, that can be verified and that are consistent. Formalization is also required for the automation of tasks such as test derivation or requirements simulation.

The main contribution of this paper is a definition of formal semantics for a textual representation of use cases. We assume the UML definition of use cases including `<< include >>` and `<< extend >>` relations. We also take use case sequencing into consideration. According to the UML, use cases should be “useful on their own”. Each use case should specify a unit of useful functionality that a system provides to its users [21]. The functionality provided by a use case is initiated by an actor and must be completed for the use case to complete. It is not always possible to structure use cases such that they are completely independent one from the other. Complex applications often involve several tasks with a high degree of independence, but that need to be synchronized in different ways. As an example, we present an “Online Broker System” in Section 3. Intuitively, an *order* needs to be submitted before *suppliers* can bid. Different policies can then be envisioned for handling bids. For instance, the application requirements might ask for all bids to be received first before one is selected, or a selection may be made as bids are received. It is not considered a good practice to use UML relationships to structure use cases in a way that these types of sequencing constraints are obtained. The resulting use case models suffer from the functional decomposition problem [7] that makes use cases difficult to maintain and pushes implementation toward non-object-oriented paradigms.

We chose Basic Petri nets [22] to express use case execution semantics. This choice is motivated by the possibility to use the same formalism for the execution semantics of both single and sequentially related use cases. Petri nets allow modeling of state based concurrent systems and are well suited to the specification of synchronization issues. They are supported by a variety of analysis and simulation tools. The Petri nets semantics are used as a basis for state model synthesis from use cases. One of our goals in doing so is to validate the defined semantics. Another objective is to provide a translation from textual use cases to an UML-based graphical representation. The generated state models capture single use case behavior as UML *StateCharts*, and use cases sequential relations as UML *Activity Diagrams* [21]. A state model obtained from related use cases integrates behaviors defined separately in a single graphical model that can be simulated and analyzed for consistency and completeness.

The remainder of this paper is organized as follow. We discuss some related works in the next section. In section 3, we introduce use case models and present an abstract and a concrete syntax for textual use cases. Use case syntax is based on our previous works in [25, 27]. The Petri nets semantics of use cases are presented as a set of Mapping Rules in

section 4. These semantics are used in section 5 for the synthesis of state models from use cases. Finally we conclude the paper in section 6.

## 2 Related works

Different works related to use cases are based on a variety of use case notations with formal semantics. In [23, 16, 24] use cases are defined in relation to formal pre/postconditions. The formalization allows derivation of conceptual models in [16], and test generation in [24]. Interaction models including UML Sequence Diagrams [21], Message Sequence Charts (MSCs) [9] and Live Sequence Charts (LSCs) [4] are frequently used formalisms for use cases description. Each use case is considered as a set of related scenarios, and these scenarios are elaborated using interactions. There are well defined semantics for the common notations for interactions such as UML Sequence Diagrams, MSCs and LSCs. The main problem tackled by interaction-based approaches concerns scenarios relation within and between use cases. Most of the scenario-based approaches use formal state models as a representation of the integrated behavior defined by related scenarios. A recent survey listed 21 such approaches [17]. Proposed solutions for relating scenarios include: annotation of scenarios with state labels [14, 13, 28], the use of a high-level representation of sequencing relations between scenarios [8, 15, 28, 33, 32], and operation predicates [26, 31].

In our work, we represent use cases using a restricted form of the natural language. Other automated approaches based on restricted natural language include [18, 19]. In [18], use cases adhering to defined language structures are parsed to automatically extract information for UML class and interaction diagrams generation. Specific syntactic forms are also assumed for use cases in [19]. A statistical parser is used to identify the different types of actions making up use case steps, and an executable regular expression representation of the use case is produced. There no explicit mention of operational semantics in [18, 19]. It can be deduced from the synthesized models that control flow semantics are assumed. The basic operational semantics considered for use cases in this paper is similar. Our goal in this paper is to explicitly state these semantics. In addition, unlike [18, 19], we consider UML use case inclusion and extension relationships as well as use case sequencing constraints.

This paper builds from previous works of ours where we defined an abstract syntax and a concrete syntax for use cases [25], and proposed use cases sequencing constructs [27]. The abstract syntax is presented as a meta-model and the concrete syntax is a restricted form of natural language. These previous results are summarized in section 3 in part in order to ensure this paper is self-contained, but also to account for updates to the use case syntax. In [25], we presented an algorithm for StateChart generation from use cases. Differently to this paper, StateChart generation in [25] is based on a formal description of operations using *added/withdrawn* predicates. In the present work our focus is on control flows semantics of use cases. State model synthesis is performed without the need for operations specification.

This paper discusses StateChart generation from Petri nets. A work related to this matter is presented in [5], where a structure preserving algorithm that translates Petri nets to StateCharts is introduced. We do not use this algorithm in part because it does not

consider compound StateChart transitions. However, we show that Petri nets obtained from “consistent” use cases satisfy the required properties listed in [5] for the obtainment of equivalent StateCharts.

### 3 Use Cases Modeling

There are two levels in use cases description according to the UML. The use case model level concerns use cases abstracted from internal details, and shows use cases relations within the system’s environment and each with the other. The second level concerns the description of use cases internal interactions.

#### 3.1 Use Cases model

A UML use case model consists of use cases, actors and relationships. We formally define a use case model as a tuple  $[Act, Uc, Rel, InitialUc]$  with:  $Act$  a set of actors,  $Uc$  a set of use cases,  $Rel = Rel_{actuc} \cup Rel_{inc} \cup Rel_{ext}$  a set of relations and  $InitialUc \subset Uc$  a set of *initial* use cases. Actors are entities in the domain model that interact with the system. Use cases are descriptions of these interactions. Each use case is initiated by an actor and is related to the achievement of a goal of interest to this actor or other actors in the system. The set of relations  $Rel$  includes relationships between actors and use cases ( $Rel_{actuc}$ ) and relationships between use cases. The former are defined on domain  $Act$  and range  $Uc$ . Relationships between use cases include  $\ll include \gg$  relationships ( $Rel_{inc}$ ) and  $\ll extend \gg$  relationships ( $Rel_{ext}$ ). An  $\ll include \gg$  relationship  $uc_{base} \times uc_{inc}$  denotes the inclusion of use case  $uc_{inc}$  as a sub-process of use case  $uc_{base}$  (the *base use case*). An *extend* relationship  $uc_{ext} \times econd \times epoints \times uc_{base}$  denotes an extension of a use case  $uc_{base}$  as addition of “chunks” of behaviors defined in an *extension use case*  $uc_{ext}$ . These chunks of behaviors are included at specific places in the base use case called *extension points* ( $epoints$ ). Each extension is realized under a specific condition ( $econd$ ).

Figure 1 shows an example of use case diagram in the UML notation. The system under consideration is an *Online Broker System*. The goal of the system is to allow customers to find the best supplier for a given order. A customer fills up an online order form and after submission, the system broadcast it to suppliers. We assume three suppliers in this example, “SupplierA”, “SupplierB” and “SupplierC”. Each supplier after examining the order may decide to decline or submit a bid. Submitted bids are sent back to the broker to be shown to the customer, who eventually asks the system to proceed with a bid. The use case model corresponds to a tuple  $[Act, Uc, Rel, InitialUc]$  with:

- $Act = \{ \text{“Customer”}, \text{“SupplierA”}, \text{“SupplierB”}, \text{“SupplierC”}, \text{“Payment System”} \}$ .
- $Uc = \{ \text{“Submit order”}, \text{“Process Bids”}, \text{“Register Customer”}, \text{“Handle Payment”}, \text{“SupplierA bid for order”}, \text{“SupplierB bid for order”}, \text{“SupplierC bid for order”}, \text{“Cancel Transaction”} \}$ .

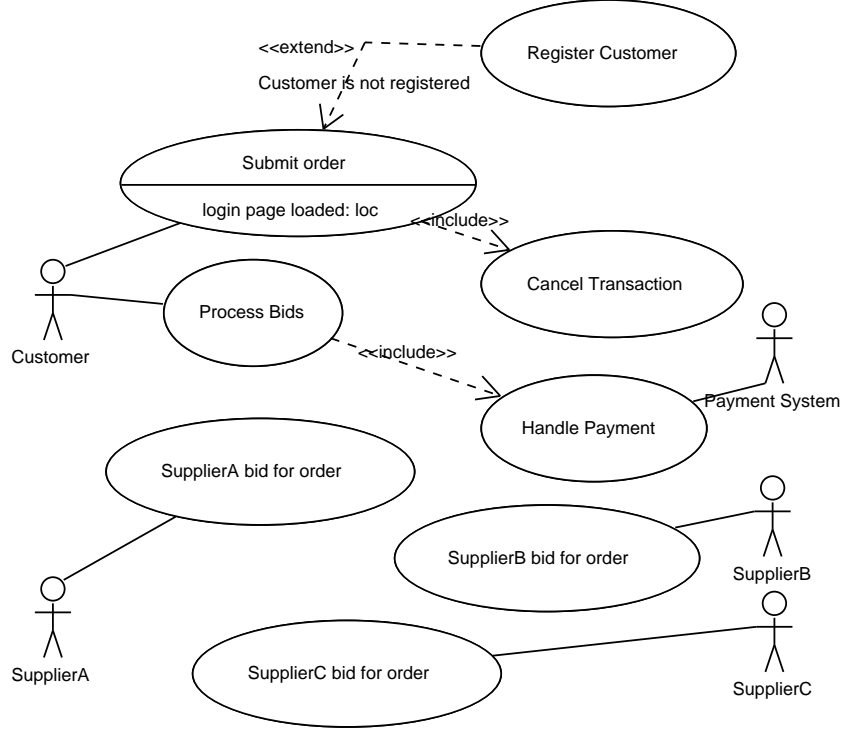


Figure 1: Example of UML Use Case diagram for an Online Broker System.

- A set of relations  $Rel = Rel_{actuc} \cup Rel_{inc} \cup Rel_{ext}$ .

Relationships between actors and use cases  $Rel_{actuc} = \{ \text{"Customer"} \times \text{"Submit order"}, \text{"Customer"} \times \text{"Process Bids"}, \text{"SupplierA"} \times \text{"SupplierA bid for order"}, \text{"SupplierB"} \times \text{"SupplierB bid for order"}, \text{"SupplierC"} \times \text{"SupplierC bid for order"}, \text{"Payment System"} \times \text{"Handle Payment"} \}$ .

The set of include relationships  $Rel_{inc}$  is  $\{ \text{"Process Bids"} \times \text{"Handle Payment"}, \text{"Submit order"} \times \text{"Cancel Transaction"} \}$ .

The set of extend relationships  $Rel_{ext}$  is  $\{ \text{"Register Customer"} \times \text{not}(\langle \text{Customer}, \text{registered} \rangle) \times \{ep1\} \times \text{"Submit order"} \}$ .  $\text{not}(\langle \text{Customer}, \text{registered} \rangle)$  represents the condition under which the extension takes place (we define conditions in section 3.2.3).  $ep1$  is an extension point defined in reference to use case *Submit order*. We define extension points in section 3.2.1.

- $InitialUc = \{ \text{"Submit order"} \}$ , assuming use case "Submit order" is enabled by default.

We define *main use cases* as the subset of use cases in a use case model that are not including use cases nor extension use cases.

**Definition 1** Given a use case model  $\mathcal{M} = [Act, Uc, Rel = Rel_{actuc} \cup Rel_{inc} \cup Rel_{ext}, InitialUc]$ ,  $UC_{main}$  the set of main use cases of  $\mathcal{M}$  is such that

$$UC_{main} = \{uc \in Uc \mid \nexists r_i = uc_x \times uc \in Rel_{inc} \text{ and } \nexists r_j = uc \times econd \times epoints \times uc_y \in Rel_{ext}\}.$$

All initial use cases must be *main* use cases for consistency.

**Consistency Rule 1** *Any use case model  $\mathcal{M} = [Act, Uc, Rel, InitialUc]$  with a set of main use cases  $UC_{main}$  must be such that  $InitialUc \subset UC_{main}$ .*

Use case diagrams are abstract high-level view of functionality. They do not describe interactions. In the next section, we discuss use cases description. We defined an abstract syntax for use case interactions inspired from Cockburn’s template [3]. We also developed a *restricted* form of natural language for concrete representation of use cases.

## 3.2 Use cases description

Figure 2 shows a UML metamodel for use case description. This metamodel defines an abstract syntax for use cases description as an extension to the UML framework. We specialized class *UseCase* (defined in the UML Specification UseCase package [21]) into *NormalUseCase* and *ExtendUseCase* to account for their differences in description. A *normal use case* defines complete behaviors. Its execution results on either the fulfillment of a goal or an error situation. An *extend use case* specifies a set of behavior *chunks* intended to extend the behavior defined by other use cases. The distinction between normal and extend use cases also allows a more rigorous definition of the  $\ll include \gg$  relation by enforcing that *UseCaseInclusion* can only refer to a *NormalUseCase*.

### 3.2.1 Normal use cases

According to our metamodel, a normal use case description is a tuple  $[UCTitle, UCPrec, UCFoll, UCSt, UCAlt, UCPost]^1$  with:

- *UCTitle* a label that uniquely identifies the use case, *UCPrec* a *Constraint* that must be true before an instance of the use case can be executed (the term *constraint* is used in the UML specification to refer to conditions),
- *UCFoll* a possibly null *follow list* specifying which use cases must precede the described use case and how these use cases are synchronized [27],
- *UCSt* a sequence of *use case steps* (*main steps sequence*),
- *UCAlt* a set of *global alternatives* that apply to all the steps in the use case, and
- *UCPost* a condition that must be true at the end of the use case *main scenario* (*success postcondition*).

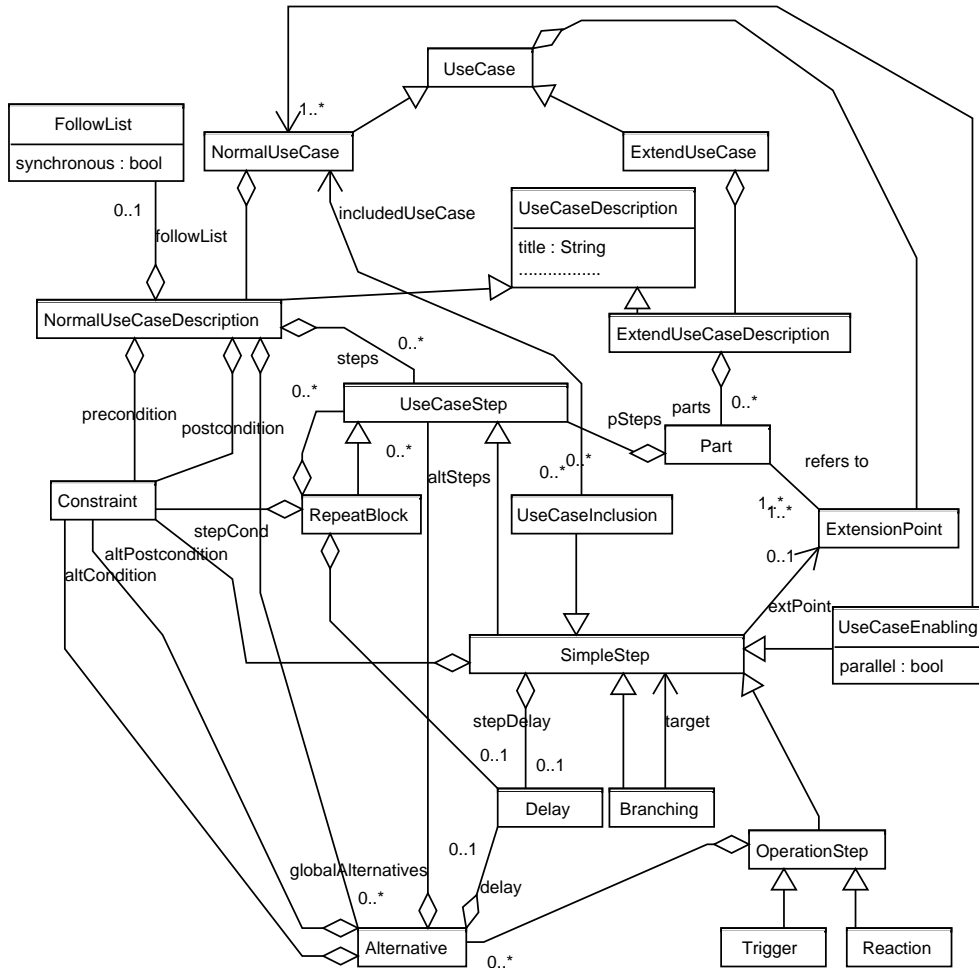


Figure 2: UML metamodel for use cases description.

Figure 3 shows a description of use case “Submit order”. The remaining normal use cases in the “Online Broker System” example are shown in the Appendix.

A non null *follow list* lists one or more use cases that are “followed” by the described use case. For instance, after a Customer has submitted an order, the Broker System broadcasts it to the suppliers (*SupplierA*, *SupplierB* and *SupplierC* in this case). Therefore use cases *SupplierA Bid*, *SupplierB Bid* and *SupplierC Bid* each follow use case *Submit order*. The *follow list* of these use cases is used to capture these sequencing requirements. When multiple use cases are listed in a *follow list*, these use cases are specified as an expression reflecting how they should be synchronized in relation to the described use case. We use the two

<sup>1</sup>Note that other elements are included in the use case template. However, only the elements above are relevant to this paper.

Title: Submit order

System Under Design: Broker System

Precondition: The Broker System is online and the Broker System welcome page is being displayed

Success Postcondition: An Order has been broadcasted

#### STEPS

1. The Customer loads the login page
2. The Broker System asks for the Customer's login information
3. The Customer enters her login information
4. The Broker System checks the provided login information
5. IF The Customer login information is accurate THEN The Broker System displays an order page
6. The Customer creates a new Order
7. Repeat while the Customer has more items to add to the Order
  - 7.1. The Customer selects an item
  - 7.2. The Broker System adds the selected item to the order
8. The Customer submits the Order
9. The Broker System broadcast the Order to the Suppliers
10. enable in parallel use cases SupplierA bid for order, SupplierB bid for order, SupplierC bid for order

#### ALTERNATIVES

- \*1.
  - \*1a. The Customer selects cancel operation
  - \*1b. The Broker System displays the welcome page
- 2a. after 60 seconds
  - 2a1. The Broker System displays a login timeout page
- 4a. The Customer login information is not accurate
  - 4a1. GOTO Step 2.
- 8a. The Order is empty
  - 8a1. The Broker System displays an error page

#### EXTENSION POINTS

- STEP 1. login page loaded



operators AND and OR with the following meaning. Operator AND expresses synchronization (*synchronized follow list*) while OR captures asynchronism (*unsynchronized follow list*). More formally, given use cases  $uc_0, uc_1, uc_2, \dots, uc_n$ , the following interpretation is given.

- If the *follow list* of  $uc_0$  is specified as “ $uc_1$  AND  $uc_2$  AND  $\dots uc_n$ ”, all of  $uc_1, uc_2, \dots, uc_n$  must reach a point from which use case  $uc_0$  is *enabled* before use case  $uc_0$  (synchronism).
- If the *follow list* of  $uc_0$  is specified as “ $uc_1$  OR  $uc_2$  OR  $\dots uc_n$ ”, use case  $uc_0$  may be executed as soon as any of use cases  $uc_1, \dots, uc_n$  reaches a point from which use case  $uc_0$  is *enabled* (asynchronism).

As an example, the *follow list* of use case “Process Bids” described in Figure 22 is *unsynchronized*. A Customer may proceed with a bid as soon as a supplier has replied without waiting for other replies.

### 3.2.2 Extension use cases

An extension use case includes one or more *parts*. These parts are inserted at specific *extension points* in a *base use case* as realization of an *extend* relationship. An extension use case is formally a tuple [ $UCTitle, Parts$ ] with:  $UCTitle$  as previously defined and  $Parts$  a set of parts. Each part is a tuple [ $ExtPoint, Steps$ ].  $ExtPoint$  is a reference to an extension point and  $Steps$  a sequence of steps (a *part steps sequence*). As an example, Figure 4 shows an extension use case titled *Register Customer* with one part. The use case diagram in Figure

<p>Title: Register Customer</p> <p>EXTENSION USE CASE PARTS</p> <p>PART 1. At Extension Point login page loaded</p> <ol style="list-style-type: none"> <li>1.1. Customer selects registration operation</li> <li>1.2. Broker System asks for Customer name, date of birth and address</li> <li>1.3. Customer enters registration information</li> <li>1.4. Broker System validates Customer information</li> <li>1.5. Broker System generate login information for Customer</li> </ol> <p>ALTERNATIVES</p> <ol style="list-style-type: none"> <li>1.4.a. Customer registration information is not valid <ol style="list-style-type: none"> <li>1.4.a.1. Broker System displays registration failure page</li> </ol> </li> </ol>
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Figure 4: Extension use case “Register Customer”.

1 defines an extend relationship between *Register Customer* and the base use case *Submit*

order. The meaning of the relationship is that use case *Register Customer* extends use case *Submit order* when condition “*Customer is not registered*” holds.

### 3.2.3 Conditions

Each condition is formally a *predicate*, the negation of a predicate, conjunction of predicates or disjunction of predicates. A predicate is a pair  $\langle E, V \rangle$  where  $E$  is an *entity* and  $V$  a *value*. Entities refer to *concepts* (actors or the system under consideration) or *attributes of concepts*. For instance use case *Submit order* precondition is formally the conjunction of predicates  $\langle \text{“Broker System”, “online”} \rangle$  and  $\langle \text{“Broker System”. “welcome page”, “displayed”} \rangle$ . In our concrete syntax [25], predicates are represented in the form “*name of entity*” “*verb*” “*possible value of entity*” with “*verb*” a conjugated form of a limited number of verbs including **to be** and **to have**. A *domain model* that enumerates all the entities in the application and their possible values is needed for parsing. A substantial part of this model is obtained by pre-processing use cases [20].

### 3.2.4 Use case steps

Use case steps include *simple steps* and *repeat blocks*. A repeat block denotes an iterative execution of a sequence of use case steps (*repeat steps sequence*) according to a condition and/or a time delay. Repeat blocks are introduced with keywords **repeat**, **while** and **until**. For instance, step 7 of use case *Submit order* is a repeat block with two steps that are iterated according to a condition. More formally, we define a repeat block as a tuple  $[RGuard, RDelay, RSteps]$  with  $RGuard$  a possibly null *guard* condition,  $RDelay$  a possibly null *delay* and  $RSteps$  a sequence of steps.

We distinguish the following types of simple steps: *operation steps*, *branching statements*, *use case inclusion directives* and *use case enabling directives*. A simple step may be constrained by *explicit* and *implicit guards* and/or a *delay*. A *guard* is a condition that must hold for the step to be possible. We introduce explicit guards using keywords **if .. then**. As an example, step 5 in use case *Submit order* is constrained by an explicit guard. In addition to explicitly specified *guards*, a step  $st_i$  is also constrained by implicit guards that include the negation of the conjunction of all the conditions of the step  $st_{i-1}$  alternatives (if any). Step  $st_{i-1}$  being a step immediately preceding  $st_i$  in the same steps sequence. For instance, although step 9 in use case “Submit order” does not include an explicit guard, it includes condition “the order is NOT empty” as implicit guard (the negation of alternative *8a* condition). A delay specifies a minimum time amount that must pass before a step is possible. Delays are counted from the completion moment of the previous step or from when the use case became enabled when applied to the first step of a use case. We introduce delays using keyword **after**.

An *operation step* denotes the execution of an operation by an actor in the environment of the system (a *trigger*) or the system itself (a *reaction*). Steps 1 and 3, 6, 7.1, and 8 in use case *Submit order* correspond to triggers while steps 2, 4 and 5, 7.2 and 9 correspond to reactions. Triggers and reactions are distinguished based on information in the domain

model. Our concrete syntax [25], also assume that operations are declared in the domain model according to the format “*action\_verb* [*action\_object*]”. Where the *action\_verb* is a verb in infinitive and the *action\_object* refers to a concept or an attribute of a concept affected by the action. As an example, “load login page” is an operation name where the action verb is “load” and the action object is “login page”. Given this naming convention, an *operation step* has the following form:

“*name of concept*” “*action\_specification*” [“*preposition*” “*action\_participant*”]<sup>2</sup>

The “*action\_specification*” has the form

“*conjugated\_action\_verb*” [“*action\_object*”]

The “*conjugated\_action\_verb*” is the “*action\_verb*” used in the concept operation declaration in the present tense.

An operation step may be associated with an *extension point*; a label that references a particular point in a use case where interactions defined in extension use cases may be inserted. An extension point corresponds to the point of execution reached after a successful completion of the operation. Step 1 of use case “Submit order” is associated with an extension point labelled “*login page loaded*”. An operation step may also be associated with one or more *alternatives*. An alternative specifies a possible continuation of a use case after a step. Alternatives are used to describe exceptions, error situations or less common courses of events. Formally an alternative is a tuple [*Acond*, *Adelay*, *Asteps*, *Apost*] with *Acond* a constraint that must be true for the alternative to be possible, *Aftd* a delay, *Asteps* a sequence of use case steps (*alternative steps sequence*) and *Apost* an alternative postcondition. Our use case notation allows *global alternatives*. For instance, use case “Submit order” includes a global alternative labelled “\*1”. A global alternative *galt* is equivalent to the inclusion of *galt* to each operation step where *galt* is possible. In use case “Submit order”, alternative \*1 effectively only applies to steps corresponding to *triggers*, because its condition (*Acond*) and delay (*Adelay*) are both null.

A *branching statement* includes a reference to a step  $st_i$  such that the flow of use case events continues from  $st_i$  whenever the *branching statement* is interpreted. We refer to  $st_i$  as the branching statement *target*. Step 4a1 in use case “Submit order” branches to step 2. Execution of that step would result in the flow of events continuation from step 2.

A *use case inclusion directive* is a realization of an *include* relationship between the use case and an included use case referred in the directive. As an example, step 3 in use case “Process Bids” is an inclusion directive referring to use case “Handle Payment” shown in Figure 23.

A *use case enabling directives* allows to explicitly state control flow between use cases. The directive can refer to one or more use cases. In the latter case, the directive may specify whether the enabled use cases execute concurrently (with keyword in **parallel** or alternatively. For instance, step 10 in use case “Submit order” is an enabling directive for use cases “SupplierA bid”, “SupplierB bid” and “SupplierC bid”. These three use cases may

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<sup>2</sup>Elements between “[]” are optional.

execute concurrently. Only one use case may execute in the non-parallel variant of the use case enabling directive. As soon as a use case start executing, all other use cases referred to in the directive are considered disabled. For consistency we assume the first events of the enabled use cases are distinct enough that an unambiguous choice can be made among the use cases. More formally, suppose `firstEvent` is a function such that given a use case  $uc$  `firstEvent(uc)` is the first event of  $uc$ , the following Consistency Rule must be satisfied.

**Consistency Rule 2** *The set  $UC_{edir} = \{uc_1, \dots, uc_n\}$  of use cases referred to in a non-parallel **enabling** directive  $edir$  must be such that  $\forall uc_i \in UC_{edir}, \nexists uc_j (i \neq j) \in UC_{edir}$  such that  $firstEvent(uc_i) = firstEvent(uc_j)$ .*

A use case may enable itself and there is no restriction to where the directive may appear. Notice that *follow lists* and use case enabling directives depend each on the other. The following rule must be satisfied for consistency.

**Consistency Rule 3** *When a use case  $uc_i$  refers to a use case  $uc_j$  in its follow list, use case  $uc_j$  must include an enabling directive referring to use case  $uc_i$ . Conversely, for each use case referred to in an enabling directive, the enabled use case must include the enabling use case in its follow list.*

A use case enabling directive may be followed by subsequent steps. These steps are assumed to execute concurrently with the enabled use cases.

### 3.2.5 Steps Sequences

A *steps sequence* is a succession of steps in a use case. A normal use case [ $UCTitle, UCPrec, UCFoll, UCSt, UCAlt, UCPost$ ] includes a *main steps sequence*  $UCSt$ . The sequence of steps  $Asteps$  in an alternative [ $Acond, Adelay, Asteps$ ] is an *alternative steps sequence*. We also distinguish *repeat steps sequences* ( $RSteps$  in a repeat block [ $RGuard, RDelay, RSteps$ ]) and *part steps sequences* ( $Steps$  in an extension use case part [ $ExtPoint, Steps$ ]).

A main use case must be initiated by an actor [21]. Consequently the following consistency rule needs to be satisfied.

**Consistency Rule 4** *The first step in a main steps sequence must be a trigger.*

For consistency, a *branching statement* must not connect unrelated steps sequences. We define a *subordinate* relation `sub` between steps sequences as follow.

**Definition 2** *A steps sequence  $sep'$  is subordinate of a steps sequence  $seq$  ( $sep' \text{sub} seq$ ) if:*

- $seq$  includes a step  $st$  such that  $sep'$  is a steps sequence of an alternative of  $st$ ,
- $seq$  includes a repeat block  $rb$  such that  $sep'$  is  $rb$ 's repeat steps sequence
- there is a steps sequence  $sep''$  such that  $(sep' \text{sub} sep'')$  and  $(sep'' \text{sub} seq)$ .

**Consistency Rule 5** For any branching statement  $step_o$  such that  $step_d$  is  $step_o$  target, let  $seq_o$  be a steps sequence such that  $step_o \in seq_o$  and  $seq_d$  be a steps sequence such that  $step_d \in seq_d$ ,  $seq_o$  and  $seq_d$  must be such that  $seq_d = seq_o$  or  $seq_d \text{sub } seq_o$ .

A branching statement must also be last in a steps sequence as subsequent steps are unreachable.

**Consistency Rule 6** For any steps sequence  $seq = step_0 \cdots step_n$ , if  $\exists i$  such that  $step_i$  is a branching statement, then  $i$  must be equal to  $n$ .

A normal use case may include a precondition and a set of postconditions (a success postcondition and alternative postconditions). Preconditions and postconditions are implicit specifications of use case sequencing constraints. A use case precondition is a *condition* that needs to hold before the use case, while a use case postcondition is a *condition* that is guaranteed to hold at the end of the use case. Assuming functions **pre** and **post** such that: **pre**(*uc*) is the precondition of a use case *uc* and given *seq* a *main* or *alternative* steps sequence in *uc*, **post**(*seq*, *uc*) is the postcondition associated with *seq*, use case *uc*<sub>1</sub> enables use case *uc*<sub>2</sub> (and *uc*<sub>2</sub> follows *uc*<sub>1</sub>) if there is a steps sequence *seq*<sub>*i*</sub> in *uc*<sub>1</sub> such that **post**(*seq*<sub>*i*</sub>, *uc*<sub>1</sub>)  $\Rightarrow$  **pre**(*uc*<sub>2</sub>). For instance, by analyzing the pre and postconditions of use cases in Figure 3 - 21, we can infer that use case “Submit order” could be followed by use cases “SupplierA Bid”, “SupplierB Bid” and “SupplierC Bid”. Preconditions and postconditions allows to specify which use case must precede a given use case and which use cases are enabled after a use case. However, they do not offer a possibility to specify how use cases are synchronized or whether several enabled use cases execute concurrently or alternatively [27]. Additionally, since postconditions are only checked at the end of a steps sequence, situations where enabling directives are not the last elements of steps sequences do not have an equivalent. Nevertheless, explicit constraints specified by use case sequencing constructs and implicit constraints derived from pre/postconditions must comply with the following consistency rules. We assume functions **pre** and **post** as above.

**Consistency Rule 7** Given use cases *uc*<sub>*i*</sub> and *uc*<sub>*j*</sub>, if *uc*<sub>*i*</sub> is included in the follow list of *uc*<sub>*j*</sub>, there must exist a steps sequences *seq*<sub>*k*</sub> in *uc*<sub>*i*</sub> such that **post**(*seq*<sub>*k*</sub>, *uc*<sub>*i*</sub>)  $\Rightarrow$  **pre**(*uc*<sub>*j*</sub>) (notice that in accordance with rule 3, *seq*<sub>*k*</sub> must include an enabling directive referring to use case *uc*<sub>*j*</sub>).

**Consistency Rule 8** Given use cases *uc*<sub>*i*</sub> and *uc*<sub>*j*</sub>, if there is a steps sequence *seq*<sub>*k*</sub> in *uc*<sub>*i*</sub> such that **post**(*seq*<sub>*k*</sub>, *uc*<sub>*i*</sub>)  $\Rightarrow$  **pre**(*uc*<sub>*j*</sub>), use case *uc*<sub>*i*</sub> must be included in the follow list of *uc*<sub>*j*</sub> and there must be a use case enabling directive referring to *uc*<sub>*j*</sub> in the steps sequence *sc*<sub>*i*</sub>.

## 4 Formal interpretation of use cases

A use case captures a set of event sequences. We represent these event sequences using the Basic Petri nets formalism.

## 4.1 Basic Petri nets

A Basic Petri net is formally defined as follow.

**Definition 3** A Basic Petri net (*P/T net*) is a triple  $[P, T, F]$  with:

- $P$  a finite set of places,
- $T$  a finite set of transitions,
- $F \subseteq (P \times T) \cup (T \times P)$  a flow relation such that for each transition  $t \in T$ , the input places of  $t$  (denoted  $\bullet t$ ) are all places  $p_{in}$  such that  $p_{in} \times t \in F$ , and the output places of  $t$  (denoted  $t \bullet$ ) are all places  $p_{out}$  such that  $t \times p_{out} \in F$ .

The set of use cases in a use case model  $\mathcal{M}$  corresponds to a P/T net  $Pn = [P, T, F]$ , and each main use case in  $\mathcal{M}$  corresponds to a *subnet* of  $Pn$  defined as follow.

**Definition 4** A subnet of a P/T net  $Pn = [P, T, F]$  is a P/T net  $Pn' = [P', T', F']$  such that  $P' \subset P, T' \subset T, F' \subset F$  and  $\forall t \in T', \bullet t \subset P', t \bullet \subset P'$ .

We use the terminology *use case P/T net* to refer to a subnet corresponding to a use case. The *initial place* of a use case P/T net as an input place to the first transition corresponding to the first step of the use case. Transitions correspond to events depicted by use cases and places represent states reached between these events. A step in a use case is typically mapped to a sequence of transitions. Figure 5 is a P/T net representation of the set of event sequences corresponding to use case “*Submit order*” extended with use case “*Register Customer*”. We use the corresponding step numbers as labels for transitions. **c1** is condition  $\langle Customer, registered \rangle$ , **c2** is condition  $\langle Customer.registration, valid \rangle$ , **c3** is condition  $\langle Customer.'login information', accurate \rangle$ , **c4** is condition  $\langle Customer, 'more items to add' \rangle$  and **c5** is condition  $\langle Order, empty \rangle$ .

The **token game** semantics governs P/T nets animation. Places may contain *tokens* and a transition  $t$  is said to be *enabled* if all  $\bullet t$  contain *tokens*. An enabled transition  $t$  may *fire* by removing a token from each of its input place and adding a token to each of its output place. A *marking*  $M$  is the distribution of tokens over places in a P/T net. Each marking represents a state of the described system. The *initial marking* of a P/T net corresponding to a use case model  $\mathcal{M} = [Act, Uc, Rel, InitialUc]$  is such that there is a token in all the initial places of the use cases in *InitialUc* and every other place is empty. More formally,  $M_i$  a marking of a use case model P/T net  $[P, T, F]$  is a function  $M_i : P \rightarrow \mathbb{N}$  such that  $\forall p \in P M_i(p)$  is the number of tokens in place  $p$ .  $M_0$  the *initial marking* of a use case model P/T net is defined as follow. We suppose  $initialp$  is a function such that  $initialp(uc)$  is the initial place of use case  $uc$  P/T net.

$$M_0(p) = \begin{cases} 1 & \text{if } (\exists uc \in InitialUc | initialp(uc) = p) \\ 0 & \text{otherwise} \end{cases}$$

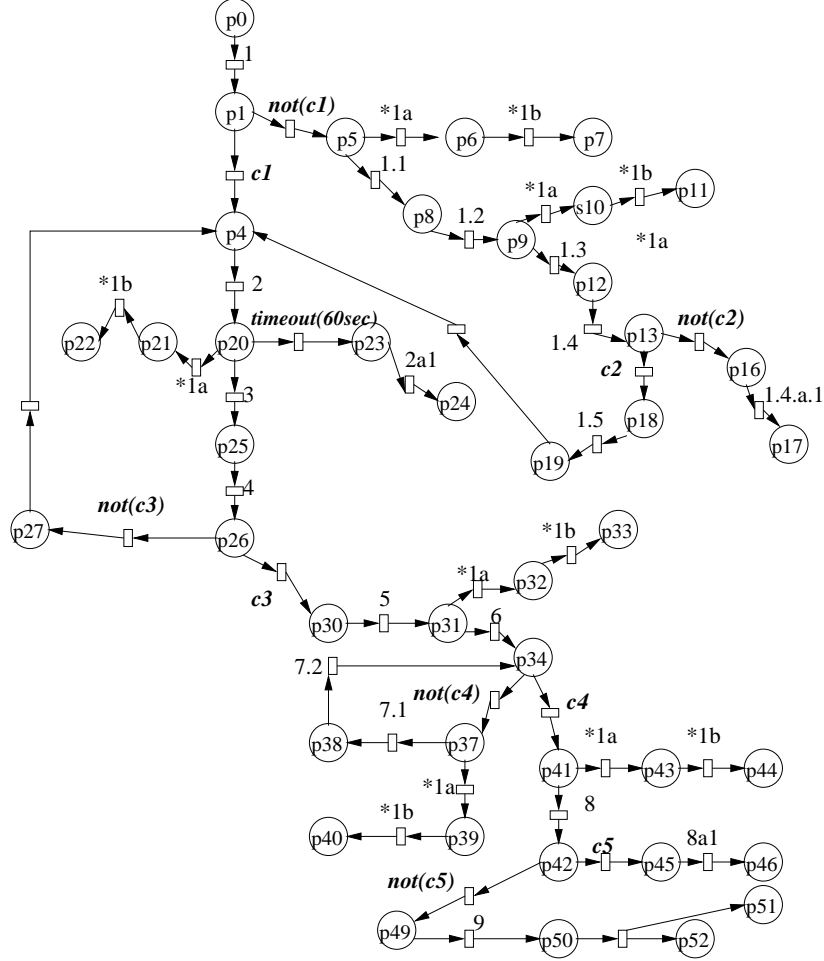


Figure 5: Petri net representation of use case “Submit order” set of possible event sequences.

Each subsequent marking  $M_i$  ( $i > 0$ ) is derived from its preceding marking  $M_{i-1}$  after the firing of an enabled transition  $t$  as follow:

$$M_i(p) = \begin{cases} M_{i-1}(p) - 1 & \text{if } (p \in \bullet t \text{ and } p \notin t \bullet) \\ M_{i-1}(p) + 1 & \text{if } (p \in t \bullet \text{ and } p \notin \bullet t) \\ M_{i-1}(p) & \text{otherwise} \end{cases}$$

Each marking ( $M_i$ ) is a possible state of the system. An execution trace is a sequence of events obtained from the sequence of transitions firing starting with a P/T net initial marking.

## 4.2 Events

Transitions in a use case P/T net correspond to the occurrence of events. We distinguish *trigger events* (corresponding to *trigger transitions*), *reaction events* (corresponding to *re-*

action transitions), *decision events* (corresponding to *decision transitions*), *timeout events* (corresponding to *timeout transitions*) and a *null event* (corresponding to *null transitions*). The set of events depicted by a use case is partially ordered. Events derived from a same steps sequence are totally ordered.

A *trigger event* denotes the execution of an operation by an actor in the environment, while a *reaction event* denotes the execution of an operation by the system under consideration. Although execution of an operation takes certain duration, we consider that an operation event is discrete and correspond to the moment of completion of the operation. A *decision event* occurs whenever one of different associated *conditions* evaluates to **true**. For instance, in Figure 5, decision events for conditions  $\langle \textit{Customer}, \textit{registered} \rangle$  and  $\text{not}(\langle \textit{Customer}, \textit{registered} \rangle)$  are possible after operation event *loads login page*. Suppose predicate  $\langle \textit{Customer}, \textit{registered} \rangle$  evaluates to **true**, the corresponding decision event would be produced and the behavior would progress along the transition labelled by that event. We assume that each place is associated with a *timer* that is started whenever a transition enters that place and stopped whenever a transition leaves the place. A *timeout event*  $\text{timeout}(d)$  is produced when a delay  $d$  elapses after a *timer* has started and hasn't been stopped.

We suppose reactive semantics similar to [6] for use case P/T nets. There is no control over when actors operations (triggers) are performed. System operations (reactions), the null event as well as decision events however, happen as soon as possible. Timeout events are delayed by a delay value such that other events may occur before they are fired. As a consequence, triggers and timeouts are ignored from places when there is at least one outgoing transition corresponding to a reaction, decision or null event. More formally, let  $\text{systTrans}$ ,  $\text{trigTrans}$ ,  $\text{timeTrans}$ ,  $\text{decTrans}$  and  $\text{nullTrans}$  be functions such that given a P/T net  $[P, T, F]$  and a place  $p \in P$ ,  $\text{systTrans}(p)$  is the set of transitions  $t_r$  such that  $p \times t_r \in F$  with  $t_r$  a reaction transition,  $\text{trigTrans}(p)$  is the set of transitions  $t_t$  such that  $p \times t_t \in F$  with  $t_t$  a trigger transition,  $\text{decTrans}(p)$  is the set of transitions  $t_c$  such that  $p \times t_c \in F$  with  $t_c$  a decision, and,  $\text{nullTrans}(p)$  is the set of transitions  $t_n$  such that  $p \times t_n \in F$  with  $t_n$  a null transition.

**Definition 5** *Given a P/T net  $[P, T, F]$ , an event  $ev$  is ignored if  $ev$  corresponds to a transition  $t$  from a place  $p$  with  $t \in \text{trigTrans}(p)$  or  $t \in \text{timeTrans}(p)$ , and  $\exists t'$  such that  $t' \in \text{decTrans}(p)$  or  $t' \in \text{nullTrans}(p)$ .*

Additionally, a timeout event with delay  $d$  is ignored from a place in presence of another timeout event with a delay less than  $d$ .

**Definition 6** *Given a P/T net  $[P, T, F]$ , a timeout event  $ev$  is ignored if  $ev$  corresponds to a transition  $t_{ev} \in \text{timeTrans}(p)$  with delay  $d$ , and  $\exists t'_{ev} \in \text{timeTrans}(p)$  with delay  $d'$  such that  $d' < d$ .*

Another consequence of the reactive semantics is that behavior depicted in a use case P/T net is *non-deterministic* when (1) more than one reaction or null transition start from a place, (2) the set of transitions starting from a place includes decision transitions mix with reaction/null transitions, (3) more than one transition corresponding to a same trigger starts from a place, or (4) more than one timeout transition starts from a place. More formally,



**Definition 7** The behavior depicted by a P/T net  $[P, T, F]$  is non-deterministic if  $\exists p \in P$ :

- (1)  $|\text{sysTrans}(p) \cup \text{nullTrans}(p)| > 1$ , or,
- (2)  $|\text{decTrans}(p)| > 0$  and  $(|\text{sysTrans}(p) \cup \text{nullTrans}(p)| > 0)$ , or,
- (3)  $\exists t_t \in \text{trigTrans}(p), \exists t'_t \in \text{trigTrans}(p) (t_t \neq t'_t)$  such that  $t_t$  and  $t'_t$  correspond to the same operation, or,
- (4)  $\exists t_t \in \text{timeTrans}(p), \exists t'_t \in \text{timeTrans}(p) (t_t \neq t'_t)$  such that  $t_t$  and  $t'_t$  correspond to timeout events with the same delay.

Notice that a P/T net such that none of (1) - (4) is satisfied may still exhibit *non-deterministic* behavior. For instance when decisions events corresponding to different transitions can hold at the same time in a place. The Mapping Rules presented in the next section produce P/T nets devoid of *non-determinism* as defined in Def. 7, when Consistency Rule 9 is satisfied (a proof to that is discussed in Section 4.4).

Let function **alternatives** be such that given a step  $step_i$ , **alternatives**( $step_i$ ) is  $step_i$  set of alternatives. Let **guard** and **delay** be two functions such that **guard**( $step_i$ ) is the conjunction of  $step_i$  explicit and implicit guards (Cf. Def. 9), and **delay**( $step_i$ ) is  $step_i$  delay. We assume **possible\_triggers** and **applicable\_delays** are two functions defined as follow. Given a step  $step_i$ , **possible\_triggers**( $step_i$ ) includes all actor operations  $astep_{j0}$  such that  $alt_j = [Acond_j = \text{null}, Adelay_j = \text{null}, Asteps_j = astep_{j0}, \dots, astep_{jm}] \in \text{alternatives}(step_i)$  and  $step_{i+1} \in \text{possible_triggers}(step_i)$  if **delay**( $step_{i+1}$ ) = **guard**( $step_{i+1}$ ) = **null** and  $step_{i+1}$  is an actor operation. Given a step  $step_i$ , **applicable\_delays**( $step_i$ ) includes all delays  $Adelay_j$  such that  $alt_j = [Acond_j = \text{null}, Adelay_j \neq \text{null}, Asteps_j] \in \text{alternatives}(step_i)$  and  $step_{i+1} \in \text{applicable_delays}(step_i)$  if **delay**( $step_{i+1}$ )  $\neq$  **null** and **guard**( $step_{i+1}$ ) = **null**.

**Consistency Rule 9** For each step  $step_i$ :

- a) there must be no  $alt = [Acond, Adelay, Asteps = astep_0, \dots, astep_m] \in \text{alternatives}(step_i)$  with  $Acond = \text{null}$ ,  $Adelay = \text{null}$  and  $astep_0$  a system operation,
- b)  $\forall trig_j \in \text{possible_triggers}(step_i), \nexists trig_k \in \text{possible_triggers}(step_i)$  with  $trig_j = trig_k$ ,
- c)  $|\text{applicable_delays}(step_i)| \leq 1$ .

Part c of Consistency Rule 9 also ensures absence of ignored timeouts events according to Def. 6.

### 4.3 Mapping of use cases to P/T nets

The event sequences defined by a set of normal use cases in a use case model  $\mathcal{M} = [Act, Uc, Rel, InitialUc]$  can be depicted as a P/T net  $Pn = [P, T, F]$  according to a set of mapping rules. It is assumed that all use cases in  $Uc$  satisfy the Consistency Rules stated through this paper. In the remainder of this Section,  $P$ ,  $T$  and  $F$  refer to the elements of the use case model  $\mathcal{M}$  P/T net.

Rule 1 specifies that each use case P/T net includes an *initial place*.

**Mapping Rule 1**  $\forall Pn_{uc} = [P_{uc}, T_{uc}, F_{uc}]$  such that  $Pn_{uc}$  is a use case  $Uc$  P/T net,  $\exists! p_0 \in P_{uc}$  with  $initialp(Uc) = p_0$ .

Each step in  $Uc$  corresponds to a sequence of transitions in  $Pn$ . We define a *step starting place* as a place from which the behavior defined by a step is considered in a use case P/T net. A *step final place* is the output place of the last transition corresponding to the step. We assume functions **startp** and **finalp** such that **startp**( $step_i$ ) is step  $step_i$  starting place and **finalp**( $step_i$ ) is  $step_i$  final place.

**Definition 8** Given a sequence of steps  $seq = step_0, \dots, step_n$  from use case  $Uc$ , if  $seq$  is a main steps sequence, **startp**( $step_0$ ) = **initialp**( $Uc$ ). For each  $step_i$  ( $i > 0$ ), **startp**( $step_i$ ) = **finalp**( $step_{i-1}$ ).

The starting place of the first step of *alternative steps sequences*, *repeat steps sequences* and *part steps sequences* are determined in Mapping Rules 4, 8 and 10 respectively. Steps final places are determined in Mapping Rules 3, 6, 7 and 8.

#### 4.3.1 Mapping of simple steps

Recall from Section 3.2.1 that a *simple step* may be constrained by implicit and/or explicit guards. Given a steps sequence  $seq = step_0, \dots, step_n$ , the set of implicit guards of  $step_i$  ( $0 < i \leq n$ ) is determined as follow. We assume **ext\_point** is a function such that **ext\_point**( $step_i$ ) returns the *extension point* associated with  $step_i$  if any or null. Given an extension point  $ep$  and a use case model  $\mathcal{M} = [Act, Uc, Rel = Rel_{actuc} \cup Rel_{inc} \cup Rel_{ext}, InitialUc]$ , **matchingExtends**( $ep, \mathcal{M}$ ) returns the set of all  $\langle\langle extend \rangle\rangle$  relations that refer to  $ep$  in  $\mathcal{M}$ . More formally, **matchingExtends**( $ep, \mathcal{M}$ ) =  $\{r_{ext} | r_{ext} = uc_{ext} \times econd \times epoints \times uc_{base} \in Rel_{ext} \wedge ep \in epoints\}$ . Function **ext\_conds** is such that **ext\_conds**( $extrels$ ) returns the set of all *extend conditions* referred to in a set of  $\langle\langle extend \rangle\rangle$  relations  $extrels$ . More formally, **ext\_conds**( $extrels$ ) =  $\{econd | \exists uc_{ext} \times econd \times epoints \times uc_{base} \in extrels\}$ .

**Definition 9** The set of implicit guards of  $step_i$  (**implicit\_guards**( $step_i$ )) is such that:

- If  $step_{i-1}$  is a repeat block  $rb = [RGuard, RDelay, RSteps]$ , **implicit\_guards**( $step_i$ ) =  $\{\neg RGuard\}$ .
- If  $step_{i-1}$  is a simple step, let the set  $AltConds_{i-1} = \{Acond | \exists Alt = [Acond, Adelay, Asteps, Apost] \in \mathbf{alternatives}(step_{i-1})\}$ . If  $AltConds_{i-1} \neq \emptyset$ , let  $cc_{alt} = \bigwedge_{cond \in (AltConds_{i-1})} \neg cond$ ,  $cc_{alt} \in \mathbf{implicit_guards}(step_i)$ .

If  $\text{extrels} = \text{matchingExtends}(\text{ext\_point}(\text{step}_{i-1}), \mathcal{M}) \neq \emptyset$ ,  
 let  $cc_{\text{ext}} = \bigwedge_{\text{cond} \in \text{ext\_conds}(\text{extrels})} \neg \text{cond}$ ,  $cc_{\text{ext}} \in \text{implicit\_guards}(\text{step}_i)$ .

**Mapping Rule 2** Given a step  $\text{step}_i$  from a use case  $\text{Uc}$ , suppose  $p_i = \text{startp}(\text{step}_i)$ ,  $\text{td}$  is a timeout transition with delay  $\text{delay}(\text{step}_i)$  (if non null) and  $\text{cg}$  is a decision transition corresponding to  $\text{guard}(\text{step}_i)$  (if non null).

- If  $\text{delay}(\text{step}_i)$  is non null and  $\text{guard}(\text{step}_i)$  is non null,  $P \supset \{p_j, p_k\}$ ,  $T \supset \{\text{td}, \text{cg}\}$ ,  $F \supset \{p_i \times \text{cg} \times p_j, p_j \times \text{td} \times p_k\}$ .
- If  $\text{delay}(\text{step}_i)$  is non null and  $\text{guard}(\text{step}_i)$  is null,  $P \supset \{p_k\}$ ,  $T \supset \{\text{td}\}$ ,  $F \supset \{p_i \times \text{td} \times p_k\}$ .
- If  $\text{delay}(\text{step}_i)$  is null and  $\text{guard}(\text{step}_i)$  is non null,  $P \supset \{p_k\}$ ,  $T \supset \{\text{cg}\}$ ,  $F \supset \{p_i \times \text{cg} \times p_k\}$ .
- If  $\text{delay}(\text{step}_i)$  is null and  $\text{guard}(\text{step}_i)$  is null,  $p_k = p_i$ .

Transitions corresponding to the remainder of  $\text{step}_i$  are added from place  $p_k$  in Mapping Rules 3, 5 and 6.

Each operation step and branching statement correspond a transition.

**Mapping Rule 3** For each operation step  $\text{step}_i$ ,  $P \supset \{p_l\}$ ,  $T \supset \{\text{op}\}$  and  $F \supset \{p_k \times \text{op}, \text{op} \times p_l\}$ . With  $\text{op}$  a transition corresponding to the execution of the operation referred in  $\text{step}_i$ . Place  $p_l$  is the final place of  $\text{step}_i$  ( $\text{finalp}(\text{step}_i) = p_l$ ).

An operation step may be associated with alternatives. Each alternative specifies how the execution may proceed after the step. As discussed in Section 4.2, *trigger* and *timeout* events are ignored when they conflict with *decision* events. Consequently some of a step alternatives may be ignored.

**Definition 10** An alternative  $\text{alt}_i = [\text{Acond}_i, \text{Adelay}_i, \text{Asteps}_i = \text{astep}_{i0}, \dots, \text{astep}_{ik}, \text{Apost}_i] \in \text{alternatives}(\text{step}_i)$  is ignored from  $\text{step}_i$  if:

- (1)  $\text{Acond}_i = \text{null}$ ,  $\text{Adelay}_i = \text{null}$ ,  $\text{astep}_{i0}$  is a trigger, and there is at least one alternative  $\text{alt}_j = [\text{Acond}_j \neq \text{null}, \text{Adelay}_j, \text{Asteps}_j, \text{Apost}_j] \in \text{alternatives}(\text{step}_i)$  or  $\text{step}_i$  is associated with an extension point  $\text{ep}_i$  such that  $\text{matchingExtends}(\text{ep}_i, \mathcal{M}) \neq \emptyset$ .
- (2)  $\text{Acond}_i = \text{null}$ ,  $\text{Adelay}_i \neq \text{null}$ , and there is at least one alternative  $\text{alt}_j = [\text{Acond}_j \neq \text{null}, \text{Adelay}_j, \text{Asteps}_j, \text{Apost}_j] \in \text{alternatives}(\text{step}_i)$ .

For instance, alternative \*1 in use case “Submit order” is ignored from steps 4 and 8 in accordance with Def. 10-(1).

**Mapping Rule 4** For each  $alt_j = [Acond, Adelay, Asteps = step_0^{alt} \dots step_m^{alt}, Apost] \in alternatives(step_i)$  such that  $alt_j$  is not ignored from  $step_i$ , let  $p_i^{alt}$  be  $finalp(step_i)$ :

- If  $Adelay$  is non null and  $Acond$  is non null,  $F \supset \{p_i^{alt} \times cg \times p_j^{alt}, p_j^{alt} \times td \times p_k^{alt}\}$ .
- If  $Adelay$  is non null and  $Acond$  is null,  $F \supset \{p_i^{alt} \times td \times p_k^{alt}\}$ .
- If  $Adelay$  is null and  $Acond$  is non null,  $F \supset \{p_i^{alt} \times cg, cg \times p_k^{alt}\}$ .
- If  $Adelay$  is null and  $Acond$  is null,  $p_k^{alt} = p_i^{alt}$ .

$startp(step_0^{alt}) = p_k^{alt}$ .

**Mapping Rule 5** For each branching statement  $step_i$  with target  $step_j$ ,  $F \supset \{p_k \times tn, tn \times p_j\}$ , with  $tn$  a null transition and  $p_j = initialp(step_j)$ .

As an example, use case “Submit order” branching statement *4a1* corresponds to flows  $p_{27} \times null, null \times p_4$  in Figure 5.

Use case inclusion directives are interpreted as follow. Let  $uc_{base}$  be a use case with an include directive  $inclUC$  and  $uc_{inc}$  a use case referred by  $inclUC$ . Suppose  $uc\_included$  is a function such that  $uc\_included(inclUC)$  returns  $uc_{inc}$ . Interpretation of directive  $inclUC$  resumes to rewriting of use case  $uc_{base}$  replacing  $inclUC$  with all the steps in  $uc\_included(inclUC)$ . We assume  $copy(Pn)$  is a copy of a P/T net  $Pn$  and  $copy(p, Pn')$  is the copy of a place  $p$  in  $Pn'$ . More formally,

**Definition 11** Given  $Pn = [P, T, F]$ ,  $copy(Pn)$  is a P/T net  $Pn' = [P', T', F']$  such that:

- $\forall p \in P, \exists p' \in P'$ , with  $p'$  a copy of  $p$  in  $Pn'$  ( $p' = copy(p, Pn')$ )
- $\forall t \in P, \exists t' \in T'$  with  $t'$  a transition that corresponds to an occurrence of the same event as  $t$ ,  $t'$  is a copy of  $t$  in  $Pn'$  ( $t' = copy(t, Pn')$ )
- $\forall p \times t \in F, \exists p' \times t' \in F'$  with  $p' = copy(p, Pn')$  and  $t' = copy(t, Pn')$
- $\forall t \times p \in F, \exists t' \times p' \in F'$  with  $p' = copy(p, Pn')$  and  $t' = copy(t, Pn')$

A use case inclusion directive corresponds to the inclusion of places and transitions in the P/T net of the including use case according to the following.

**Mapping Rule 6** For each inclusion directive  $step_i$  referring to an included use case  $uc_{inc}$ , let  $Pn_{inc} = [P_{inc}, T_{inc}, F_{inc}]$  be use case  $uc\_included(step_i)$  P/T net and  $Pn'_{inc} = copy(Pn_{inc}) = [P'_{inc}, T'_{inc}, F'_{inc}]$ .

- $P \supset P'_{inc}, T \supset T'_{inc}, F \supset F'_{inc}$
- Let  $p_0^{inc}$  be  $initialp(uc_{inc})$ ,  $T \supset \{tn\}$ ,  $F \supset \{p_k \times tn, tn \times copy(p_0^{inc}, Pn'_{inc})\}$  with  $tn$  a null transition.

- Let  $main_{inc} = step_0^{inc} \dots step_n^{inc}$  be use case  $uc_{inc}$  main steps sequence;  $finalp(step_i) = copy(finalp(step_n^{inc}), Pn'_{inc})$ .

A use case enabling directive corresponds to the forking of concurrent execution sequences. One sequence corresponds to the steps that follow the directive if any, while the other sequences correspond to the enabled use cases (Cf. Mapping Rules 12 - 14).

**Mapping Rule 7** For each use case enabling directive  $step_i$ ,  $P \supset \{p_f, p_e\}$ ,  $tn \in T$ ,  $F \supset \{startp(step_i) \times tn, tn \times p_f, tn \times p_e\}$  with  $tn$  a null transition.  $finalp(step_i) = p_f$ . We consider a relation *enable\_place* such that  $step_i \times p_e \in \text{enable\_place}$ .

For instance, step 10 in use case “Submit order” is mapped to flows  $p_{50} \times null$ ,  $null \times p_{51}$ ,  $null \times p_{52}$  in Figure 5.

### 4.3.2 Mapping of repeat blocks

A repeat block  $[RGuard, RDelay, RSteps]$  specifies a sequence of iterative steps  $RSteps$ . Each iteration depends on a guard condition  $RGuard$  and/or a time delay  $RDelay$ .

**Mapping Rule 8** Given a repeat block  $rblock = [RGuard, RDelay, RSteps = step_0^r \dots step_m^r]$ , let  $p_i$  be  $startp(rblock)$ ,  $cg$  a transition corresponding to condition  $RGuard$ , and  $td$  a transition corresponding to a timeout event with delay  $RDelay$ .

- If  $RDelay$  is non null and  $RGuard$  is non null,  $P \supset \{p_j^r, p_k^r\}$ ,  $T \supset \{cg, td\}$ ,  $F \supset \{p_i \times cg \times p_j^r, p_j^r \times td \times p_k^r\}$ ,  $finalp(rblock) = p_i$ .
- If  $RDelay$  is non null and  $RGuard$  is null,  $P \supset \{p_k^r\}$ ,  $T \supset \{td\}$ ,  $F \supset \{p_i \times td \times p_k^r\}$ .
- If  $RDelay$  is null and  $RGuard$  is non null,  $P \supset \{p_k^r\}$ ,  $T \supset \{cg\}$ ,  $F \supset \{p_i \times cg, cg \times p_k^r\}$ ,  $finalp(rblock) = p_i$ .
- If  $RDelay$  is null and  $RGuard$  is null,  $p_k^r = p_i$ .

$startp(step_0^r) = p_k^r$ .

Notice that the final and initial places of a repeat block are identical when a non-null guard condition is used. The final place is undetermined when the guard condition is null. In such a case, the repeat block models an endless iterative behavior that can only be terminated with an embedded branching statement.

### 4.3.3 Use case extension

Use case extension is a mechanism for the expansion a *base* use case with behaviors defined in *extension use cases*. An extension use case  $ExUc$  is a tuple  $[UCTitle, Parts]$ . Each part in  $Parts$  in turn is a tuple  $[ExtPoint, Steps]$

**Mapping Rule 9** An extension use case  $\text{Part pt} = [\text{ExtPoint}, \text{Steps}]$  corresponds to a part P/T net  $[P_p, T_p, F_p]$ .

**Mapping Rule 10**  $\forall \text{pt} = [\text{ExtPoint}, \text{Steps} = \text{step}_0^p, \dots, \text{step}_m^p]$  a part such that  $\text{pt}$  P/T net  $= [P_p, T_p, F_p]$ ,  $\exists p_0^p \in P_p$  with  $\text{initialp}(\text{pt}) = p_0^p$  and  $\text{startp}(\text{step}_0^p) = p_0^p$ .

An  $\ll\text{extend}\gg$  relation specifies a condition under which a use case extension is realized as well as the specific points (*extension points*) in the base use case where the behavior specified in parts are plugged in. Let functions `copy`, `matchingExtends` and `ext_conds` be as previously defined.

**Mapping Rule 11** For each  $\text{step}_i$  such that  $\text{ep}_i = \text{ext\_point}(\text{step}_i) \neq \text{null}$  and  $\text{Extrels} = \text{matchingExtends}(\text{ep}_i, \mathcal{M}) \neq \emptyset$ , let  $\text{Extrels}^2$  be the set of  $\text{Extrels}$  subsets without the empty set,  $\text{NCP}$  be the set  $\text{ext\_conds}(\text{Extrels})$  and let  $p_i$  be equal to  $\text{finalp}(\text{step}_i)$ .

For each  $\text{Erels} \in \text{Extrels}^2$ ,

- Let  $\text{CP}$  be the set  $\text{ext\_conds}(\text{Erels})$ ,  $p_i \times \text{cond} \in F$ ,  $\text{cond} \in T$  with  $\text{cond}$  a transition corresponding to condition  $\bigwedge_{(C \in \text{CP})} C \bigwedge_{(NC \in \text{NCP} - \text{CP})} \neg NC$
- For each  $\text{rel} = \text{uc}_{\text{ext}} \times \text{econd} \times \text{epoints} \times \text{uc}_{\text{base}} \in \text{Erels}$ , with  $\text{uc}_{\text{ext}} = [\text{UCTitle}, \text{Parts}]$  and  $\text{part}^r = [\text{ep}_i, \text{Steps}^r = \text{step}_0^r \dots \text{step}_m^r] \in \text{Parts}$ . Let  $Pn_r$  be a P/T net corresponding to  $\text{part}^r$  and  $Pn'_r = [P'_r, T'_r, F'_r] = \text{copy}(Pn_r)$ .

$$- P \supset P'_r, T \supset T'_r, F \supset F'_r$$

$$- F \supset \{\text{cond} \times \text{copy}(\text{initialp}(\text{part}^r))\}$$

$$- \text{If } \exists \text{step}_{i+1}, \text{tn} \in T, F \supset \{\text{finalp}(\text{step}_m^r) \times \text{tn}, \text{tn} \times \text{startp}(\text{step}_{i+1})\} \text{ with } \text{tn} \text{ a null transition.}$$

As an example, use case “Submit order” (Figure 3) is extended by the extension use case “Register Customer” (Figure 4) according to the  $\ll\text{extend}\gg$  relation “Register Customer”  $\times \text{not}(\langle \text{Customer}, \text{registered} \rangle) \times \{\text{ep1}\} \times$  “Submit order”. The corresponding P/T net in Figure 5 shows the inclusion of behaviors defined in “Register Customer” from place  $p_1$  (the final place of step 1). Transition  $c_1$  corresponds to an *implicit guard* obtained from the  $\ll\text{extend}\gg$  condition negation (see Section 4.3). The extension behavior corresponds to places  $p_5 - p_{19}$ . Place  $p_{19}$ , the *final place* of step 1.5 in “Register Customer”, is linked back to place  $p_4$  the initial place of step 2 in “Submit order”. Figure 6 shows an example where more than one of extension use case contribute at an extension point. The base use case  $\text{uc1}$  includes an extension point  $\text{ep1}$  that is referred to in two extension use cases ( $\text{ucex1}$  and  $\text{ucex2}$ ). Suppose the  $\ll\text{extend}\gg$  relations  $\text{ucex1} \times c1 \times \{\text{ep1}\} \times \text{uc1}$  and  $\text{ucex2} \times c2 \times \{\text{ep1}\} \times \text{uc1}$ . Figure 7 shows a P/T net corresponding to use case  $\text{uc1}$  with the extensions. Each combination of extension conditions is considered. In the situation where both  $c1$  and  $c2$  hold, the extensions in  $\text{ucex1}$  and  $\text{ucex2}$  occur concurrently.

Title: uc1	Title: ucex1	Title: ucex2
STEPS	PART1. At Ext	PART1. At Ext
1 ..	ep1	ep1
2 ..	1.1 ..	2.1 ..
3 ..	1.2 ..	2.2 ..
EXT POINTS		
STEP 2. ep1		
(a) base use case	(b) extension use case ucex1	(c) extension use case ucex2

Figure 6: Example for illustrate mapping of use case extension.

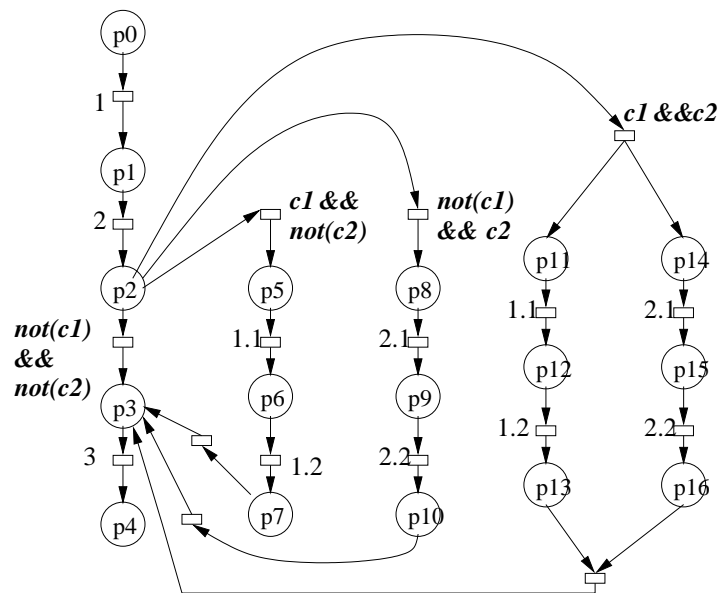


Figure 7: Petri net corresponding to use cases in Figure 6.

#### 4.3.4 Use cases sequencing

The specification of use case sequencing is based on two complementary constructs: *enabling directives* and *follow lists*. An enabling directive specifies explicitly use cases which execution might start from the point of the directive. A *parallel enabling* directive variant allows specifying concurrent execution of the enabled use cases. In the *non-parallel enabling* directive variant, a *deferred choice* [29] is assumed among the enabled use cases and only one may execute. In accordance with Consistency Rule 2, the choice is determined from the next event.

Let  $\text{isParallel}$  be a function such that given a use case enabling directive  $\text{edir}$ ,  $\text{isParallel}(\text{edir})$  returns  $\text{true}$  if  $\text{edir}$  is a *parallel enabling* directive and  $\text{false}$  otherwise, let  $\text{enabled\_uc}$  be a function such that  $\text{enabled\_uc}(\text{edir})$  is the set of use cases referred to in  $\text{edir}$ , let  $\text{places\_ucases}$  be a relation such that  $p_i$  being a place and  $uc_o, uc_d$  being use cases  $uc_o \times p_i \times uc_d \in \text{places\_ucases}$  if  $p_i$  is a place corresponding to the enabling of use case  $uc_d$  by use case  $uc_o$ , and let  $\text{edir} \times p_e \in \text{enable\_place}$  (Cf. Mapping Rule 7).

**Mapping Rule 12** *Given a use case enabling directive  $\text{edir}$  from use case  $uc_o$ ,*

- *if  $\text{isParallel}(\text{edir})$ ,  $p_e \times tn \in F$ ,  $tn \in T$  and for each  $uc_i \in \text{enabled\_uc}(\text{edir})$ ,  $p_{uci} \in P$ ,  $tn \times p_{uci} \in F$ ,  $uc_o \times p_{uci} \times uc_i \in \text{places\_ucases}$  with  $tn$  a null transition,*
- *otherwise, for each  $uc_i \in \text{enabled\_uc}(\text{edir})$ ,  $t_{ni} \in T$ ,  $p_{uci} \in P$ ,  $p_e \times t_{ni} \in F$ ,  $t_{ni} \times p_{uci} \in F$ ,  $uc_o \times p_{uci} \times uc_i \in \text{places\_ucases}$  with each  $t_{ni}$  a null transition.*

A use case may include more than one enabling directive referring to the same use case. From the point of view of the enabled use case, any of the enabling directives may lead to execution. We define a relation  $\text{unique\_places\_ucases}$  to link enabling use cases, enabled use cases and a unique place from which a transition to the enabled use case start. Let  $\text{enablePlaces}$  be a function such that  $(uc_i, uc_j)$  being a pair of *main use cases*,  $\text{enablePlaces}(uc_i, uc_j) = \{rel \mid rel = uc_i \times p_{uci} \times uc_j \in \text{places\_ucases}\}$ .

**Mapping Rule 13** *For each pair of main use cases  $(uc_i, uc_j)$ , let  $\text{EnP}$  be  $= \text{enablePlaces}(uc_i, uc_j)$ , if  $|\text{EnP}| \geq 1$ ,  $p_u \in P$ , for each  $uc_i \times p_{uci} \times uc_j \in \text{EnP}$ ,  $t_{ij} \in T$ ,  $F \supset \{p_{uci} \times t_{ij}, t_{ij} \times p_u\}$  with  $t_{ij}$  a null transition;  $uc_i \times p_u \times uc_j \in \text{unique\_places\_ucases}$ .*

Follow lists specifies how enabled use cases may execute in reference to enabling use cases. In a *synchronized follow list*, all enabling use cases must have enabled the enabled use case. An *unsynchronized follow list* describes a situation where only one enabling use case is needed. Let  $\text{isSynchronized}$  and  $\text{followed\_ucases}$  be functions such that given a use case  $Uc = [UCTitle, UCPrec, UCFoll, UCSt, UCAlt, UCPost]$ ,  $\text{isSynchronized}(UCFoll)$  is  $\text{true}$  if  $UCFoll$  is a *synchronized follow list* and  $\text{false}$  otherwise, and  $\text{followed\_ucases}(UCFoll)$  returns the list of use cases referred to in  $UCFoll$ .

**Mapping Rule 14** *Given a main use case  $Uc = [UCTitle, UCPrec, UCFoll, UCSt, UCAlt, UCPost]$ , let  $\text{FolUC}$  be  $\text{followed\_ucases}(UCFoll)$ ,*



- If  $isSynchronized(UCFoll)$ , let  $tn$  be a null transition,  $tn \in T$ ,  $tn \times initialp(Uc) \in F$ , for each  $p_{uci}$  such that  $\exists uc_i \in FolUC$  with  $uc_i \times p_{uci} \times uc \in unique\_places\_ucases$ ,  $p_{uci} \times tn \in F$ .
- If  $\neg isSynchronized(UCFoll)$ , for each  $p_{uci}$  such that  $\exists uc_i \in FolUC$  with  $uc_i \times p_{uci} \times uc \in unique\_places\_ucases$ ,  $F \supset \{p_{uci} \times t_{ni}, t_{ni} \times initialp(Uc)\}$ ,  $t_{ni} \in T$  with each  $t_{ni}$  a null transition.

As an example, Figure 8 shows three sequentially related use cases:  $uc1$ ,  $uc2$  and  $uc3$ , and Figure 9 shows a corresponding P/T net. Place  $p_2$  corresponds to the enabling directive

Title: uc1 Follows: uc2 AND uc3 STEPS 1 .. 2 enable in parallel uc2, uc3 ALTERNATIVES 1.a c1 1.a.1 .. 1.a.2 enable uc2	Title: uc2 Follows: uc1 STEPS 1 .. 2 enable uc1, uc3 3 .. 4 ..	Title: uc3 Follows: uc1 OR uc2 STEPS 1 .. 2 .. 3 .. ALTERNATIVES 2.a c2 2.a.1 .. 2.a.2 enable uc1
--	--	--

Figure 8: Example illustrating mapping of use cases sequencing.

at step 2 of use case  $uc1$ . Since it is a *parallel* directive, it is mapped to flows  $p_8 \times null$ ,  $null \times p_9$ ,  $null \times p_{10}$  denoting a forked transition. The P/T net includes  $p_{11}$  as a unique place corresponding to use case  $uc2$  enabling by use case  $uc1$ . Use case  $uc2$  enables  $uc1$  and  $uc3$  in a *non-parallel* way. Moreover, the enabling directive is followed by further steps. This translates to place  $p_{24}$  from which two transition sequences are forked to account for the parallel execution of steps 3, 4 in  $uc2$  with the enabled use cases. Two alternative flows from  $p_{28}$  reflect the *non-parallel* nature of the directive. Use case  $uc1$  *synchronized* follow list corresponds to a *join* transition to  $p_0$  from enabling use cases  $uc2$ ,  $uc3$ . In contrast, use case  $uc3$  *non-synchronized* follow list corresponds to having each enabling use case connect directly to  $p_{12}$ .

#### 4.4 Properties of P/T nets

In this Section, we establish some properties of P/T nets obtained from our Mapping Rules. These properties are needed for state model derivation as discussed in the next Section. We consider use case behavior as *non-reentrant*. That is each execution of a use case is an instantiation of behavior completely independent from other instantiations. Notice that this assumption does not prevent several simultaneous executions of a same use case. However, each invocation starts a new *activity* with its own state information. We also consider all steps following an enabling directive as constituting a separate use case. As per Mapping

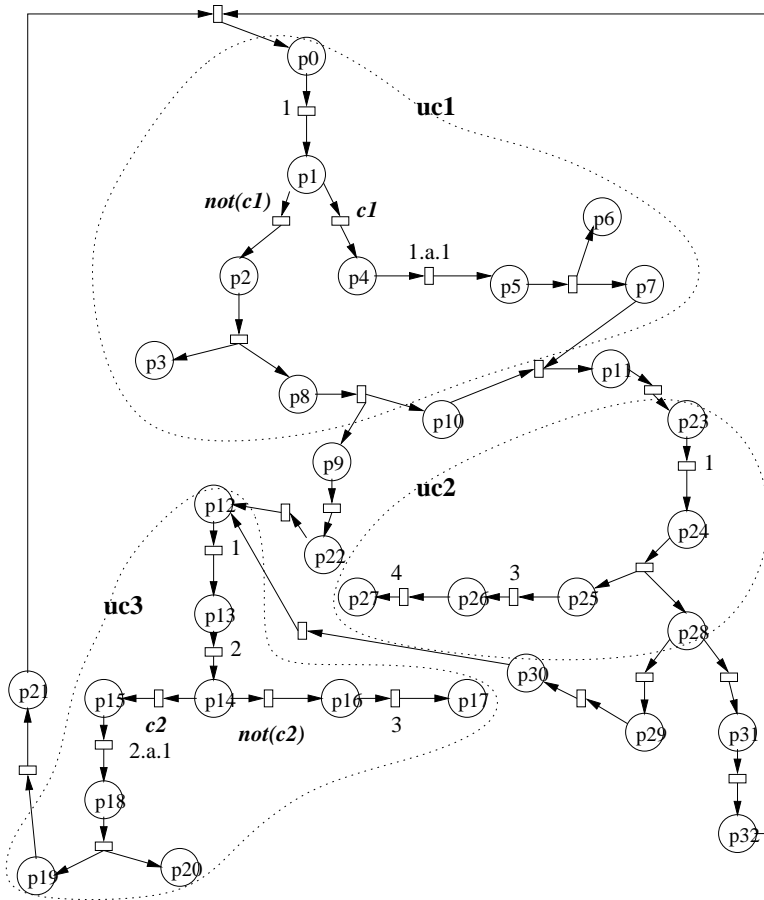


Figure 9: Petri net corresponding to use cases in Figure 8. The dotted lines delimits use case P/T nets.

Rule 7, steps after an enabling directive are concurrent with the enabled use cases. In order to adequately model such situations, a use case  $[UCTitle, UCPrec, UCFoll, UCSt = step_0, \dots, step_k, \dots, step_n, UCAlt, UCPost]$  with  $step_k$  an enabled directive and  $n > k$ , would be considered as two use cases  $[UCTitle, UCPrec, UCFoll, UCSt = step_0, \dots, step_{k-1}, UCAlt, UCPost']$  and  $[UCTitle', UCPrec', UCFoll', UCSt' = step_{k+1}, \dots, step_n, UCAlt, UCPost]$ . The following theorem follows from these assumptions.

**Lemma 1** *A use case P/T net obtained according to Mapping Rules 1 to 11 is balanced.*

*Proof:* A P/T net is balanced when all places/transitions sequences leaving a *forked* transition are subsequently *joined*. Only Mapping Rules 7 and 11 introduce *forked* transitions. One of the two forked transitions in rule 7 is not part of the use case P/T net as per the above assumptions. In the case of rule 11, balance is ensured as all the places/transitions sequences forked are disjointed and then merged before a transition links to a common output place. This situation is illustrated in Figure 7.  $\square$

**Lemma 2** *A use case P/T net obtained according to Mapping Rules 1 to 11 is such that no parallel places/transitions sequences are connected.*

*Proof:* This property is a consequence of Consistency Rule 5, which mandates that there is no *branching* from unrelated steps sequences. Parallel places/transitions sequences result from only Mapping Rule 11. The only possibility for connection between parallel places/transitions sequences would necessitate a branching statement in one *part steps sequence* with a target in another *part steps sequence*. Consistency Rule 5 prevents this.  $\square$

**Theorem 1** *A use case P/T net obtained according to Mapping Rules 1 to 11 is 1-safe.*

*Proof:* A P/T net  $Pn$  is considered *1-safe* if for all reachable marking  $M$ , each place in  $Pn$  contains at most one token. A use case P/T is a subnet restricted to a use case. According to Mapping Rule 1, there is one and only one *initial place* for a use case P/T net. Therefore, because of the *non-reentrant* assumption, only  $initialp(uc)$  is marked with a single token at the start of a use case execution (instantiation of a new use case P/T net). In order for a place  $p_k$  to contain more than one token, the use case P/T net needs to include a sequence of places/transitions  $p_1 - t_1 \dots - p_k$  such that (1)  $p_1$  is an output place of more than one transitions and (2) each of these transitions is at the end of a sequence of places/transitions starting with a *fork*. This situation is impossible as use case P/T nets are balanced (Cf. Lemma 1) and parallel sequences are unconnected (Cf. Lemma 2).  $\square$

**Lemma 3** *A use case P/T net  $[P, T, F]$  obtained according to Mapping Rules 1 to 11 is such that  $\forall p \in P$  if  $\exists p \times tn \in F$  with  $tn$  a null transition,  $\nexists t(tn \neq t)$  such that  $p \times t \in F$ .*

*Proof:* Lemma 3 is equivalent to  $(|\text{nullTrans}| = 1) \Rightarrow (|\text{systTrans}(p) \cup \text{trigTrans}(p) \cup \text{timeTrans}(p) \cup \text{decTrans}(p)| = 0)$ . This assertion is supported by the fact that transitions corresponding to the *null event* appear in a P/T net according to Mapping Rules 5, 6, 7 and 11. Because there

is no alternative or extension point associated to *branching statements*, *inclusion directives* and *enabling directives*, in each of these cases, a single transition corresponding to the *null event* is created from the departing place and this transition is the only one from that place.  $\square$

**Lemma 4** *A use case P/T net  $[P, T, F]$  obtained according to Mapping Rules 1 to 11 does not include ignored events.*

*Proof:* *Trigger* and *timeout* events are ignored when in conflict with *reaction*, *decision* or *null* events. We already established that transitions corresponding to the *null event* do not conflict with any other transitions (Cf. Lemma 3). Because of Consistency Rule 9 and because Mapping Rule 4 only considers alternatives that are *not ignored* (Def. 10) from the step under consideration, the following can be asserted  $\forall p \in P, (|\text{timeTrans}(p)| \leq 1) \wedge (|\text{trigTrans}(p) \cup \text{timeTrans}(p)| \geq 1) \Rightarrow (|\text{sysTrans}(p) \cup \text{decTrans}(p)| = 0)$ .  $\square$

**Theorem 2** *Given a use case Uc that satisfies Consistency Rule 9, the P/T net  $[P_{uc}, T_{uc}, F_{uc}]$  corresponding to Uc according to our Mapping Rules is not non-deterministic in the sense of Def. 7.*

*Proof:* None of the four criteria for *non-determinism* listed in Def. 7 is satisfied by P/T nets obtained according to the Mapping Rules.

- There is no place  $p \in P_{uc}$  such that  $|\text{sysTrans}(p) \cup \text{nullTrans}(p)| > 1$  because  $|\text{sysTrans}(p)| \leq 1$ , and either  $|\text{nullTrans}| = 0$  or  $(|\text{nullTrans}| = 1$  and  $|\text{sysTrans}(p) \cup \text{trigTrans}(p) \cup \text{timeTrans}(p) \cup \text{decTrans}(p)| = 0)$ .

$|\text{sysTrans}(p)| \leq 1$  follows from the fact that a transition  $tr \in \text{sysTrans}(p)$  if and only if step  $step_i$  in Mapping Rule 3 is an operation step and  $p$  is place  $p_k$ . Other transitions from place  $p_k$  would correspond to step  $step_{i-1}$  alternatives and extensions. All  $\langle\langle \text{extend} \rangle\rangle$  relations include a condition, and because of Consistency Rule 9, all alternatives which first step is a *system operation* have a non-null guard or delay. Therefore no other transition from  $p_k$  may correspond to a *system operation*.

$|\text{nullTrans}| = 0$  or  $(|\text{nullTrans}| = 1$  and  $|\text{sysTrans}(p) \cup \text{trigTrans}(p) \cup \text{timeTrans}(p) \cup \text{decTrans}(p)| = 0)$  follows from Lemma 3.

- For all places  $p \in P_{uc}$ ,  $|\text{decTrans}(p)| > 0 \Rightarrow |\text{sysTrans}(p) \cup \text{nullTrans}(p)| = 0$ . We already established that when  $|\text{nullTrans}(p)| > 0$ ,  $|\text{decTrans}(p)| = 0$ . We can also establish that when  $|\text{sysTrans}(p)| > 0$ ,  $|\text{decTrans}(p)| = 0$  by observing again that a transition  $tr \in \text{sysTrans}(p)$  if and only if step  $step_i$  in Mapping Rule 3 is an operation step and  $p$  is place  $p_k$ . In order for  $|\text{decTrans}(p)| > 0$ ,  $step_i$  needs to have alternatives or an extension point referred by extend relations. In all these cases, according to Def. 9,  $step_i$  would include at least one *implicit guard* and therefore,  $tr$  would not start from  $p_k$ .
- Because of Consistency Rule 9,  $\forall t_t \in \text{trigTrans}(p), \exists t'_t \in \text{trigTrans}(p) (t_t \neq t'_t)$  such that  $t_t$  and  $t'_t$  correspond to the same operation, and  $\forall t_t \in \text{timeTrans}(p), \exists t'_t \in \text{timeTrans}(p) (t_t \neq t'_t)$  such that  $t_t$  and  $t'_t$  correspond to timeout events with the same delay.  $\square$

## 5 State model synthesis

We developed an approach for state model synthesis from use cases based on the use case formalization discussed above. Given a use case model, we generate a two-level state model description of the behavior of the system under consideration. At the use case model level, the system’s behavior is seen as an UML *activity diagram* [21] with main use cases as elements. Behaviors corresponding to each main use case is detailed using a hierarchical state machine (*StateChart*). We use the term *StateChart-Chart* to refer to the graphical depiction of the two-level state model description. As discussed in the introduction, one of our motivations is to provide a visual model of complex system behavior that can be analyzed and validated. At the use case model level, it should be possible to visualize control flow between use cases without delving into details. However, the specific points from which control flows from a use case in another use case should be identifiable when needed. We present a prototype tool allowing such capabilities in Section 5.3.

### 5.1 StateChart Generation

#### 5.1.1 StateChart model

We generate a subset of UML StateChart that is formally defined as a tuple  $[Trig_c, Reac_c, G_c, S_c, Ch_c, Fk_c, Jn_c, F_c]$ .

- $Trig_c$  is a set of *triggers* consisting of operations from the environment and timeouts.
- $Reac_c$  is a set of *reactions* that are operations executed by the system.
- $G_c$  is a set of *guard* conditions.
- $S_c$  is a set of states.
- $Ch_c$  is a set of *choice pseudostates*.
- $Fk_c$  is a set of *fork pseudostates*.
- $Jn_c$  is a set of *join pseudostates*.
- $F_c$  is an edge<sup>3</sup> function.

$V_c = S_c \cup Ch_c \cup Fk_c \cup Jn_c$  is the StateChart set of *vertices*. We distinguish *simple* states, *composite* states and *orthogonal* states. We assume boolean functions *issimple*, *iscomposite* and *isorthogonal* such that given a state  $s$ , *issimple*( $s$ ) returns **true** if  $s$  is a *simple* state (**false** otherwise), *iscomposite*( $s$ ) returns **true** if  $s$  is a *composite* state (**false** otherwise) and *isorthogonal*( $s$ ) returns **true** if  $s$  is an *orthogonal* state (**false** otherwise).

---

<sup>3</sup>We use the term “edge” for StateCharts rather than “transition” to avoid confusion with Petri nets “transitions”.

- Each composite state  $s_c$  is a tuple  $[S_c, S0_c, Ch_c, Fhk_c, Jhn_c]$  with  $S_c$  a set of states,  $S0_c \in (S_c \cup Ch_c)$  a composite state initial vertex,  $Ch_c$  a set of choice pseudostates,  $Fhk_c$  a set of fork pseudostates and  $Jhn_c$  a set of join pseudostates. We assume functions **sub-vertices** and **initial\_vertex** defined as follow. **sub-vertices**( $s_c$ ) is a set of  $s_c$  *sub-vertices*. For each vertex  $s_j \in \text{sub-vertices}(s_c)$ ,  $s_c$  is a *sup-vertex* of  $s_j$ . **initial\_vertex**( $s_c$ ) returns the *initial sub-vertex* of  $s_c$ .
- For each  $s_i \in S_c$  such that **isorthogonal**( $s_i$ ), we assume function **regions** such that **regions**( $s_i$ ) is a set of parallel *regions*. Each region  $r$  is a *composite* state.

Each state is associated with a *timer* to count elapsed time while the system is waiting in that state. We assume a function **timer** such that given a state  $s$ , **timer**( $s$ ) returns the timer associated to  $s$ . Implicitly **timer**( $s$ ) is *started* whenever an edge produces a state change to  $s$ , and **timer**( $s$ ) is implicitly *stopped* whenever an edge exits from state  $s$ . A *timeout* event **timeout**( $d$ ) occurs when a timer has been started and not stopped before delay  $d$  elapsed.

Each edge is a relationship  $v_s \times \text{guards} \times \text{trig} \times \text{reacs} \times v_t$  with  $v_s \in V_c$  a *source vertex*,  $v_t \in V_c$  a *target vertex*,  $\text{reacs} \subset \text{guards}$  a set of guard conditions,  $\text{trig} \subset \text{Trig}_c$  a set of triggers and  $\text{reacs} \subset \text{Reac}_c$  a set of reactions. The compound notation  $v_s \xrightarrow{[\text{guards}]\text{trig}/\text{reacs}} v_t$  is used for edges. The UML specification defines different constraints on edges [21]. Edges with *choice pseudostates* as sources must have *trig* empty. *Fork pseudostates* are used to split an edge into several edges. The target of each of these edges must be a state in a different region of an orthogonal state. *Join pseudostates* are used to merge several edges from different regions of an orthogonal state into a single edge. An edge to a composite state  $s_c$  is equivalent to an edge ending in the initial vertex of  $s_c$  (**initial\_vertex**( $s_c$ )). An edge to an orthogonal state  $s_o$  is equivalent to an edge ending in the initial vertices of each of  $s_o$  regions. An edge starting from a composite or orthogonal state  $s$  on a trigger  $t$  is equivalent to a set of edges from each substates of  $s$  with no outgoing edge on  $t$ .

### 5.1.2 StateChart generation approach

Figures 10 - 12 show an algorithm for StateChart generation from a use case, and Figure 13 shows a StateChart generated from use case “Submit order” described in Figure 3. The StateChart generation algorithm relies on the properties established in Section 4.4. It has been shown in [5] that P/T nets which are *balanced*, *1-safe* and without connection between parallel places/transitions sequences, have structure-preserving equivalent StateCharts as a one-to-one correspondence between P/T nets places and StateChart states can be made. We also established that use case P/T nets do not include ignored events (Cf. Lemma 3) and are not non-deterministic (Cf. Theorem 2). These properties allow generation of stateCharts edges in  $v_s \xrightarrow{[\text{guards}]\text{trig}/\text{reacs}} v_t$  form.

StateChart generation starts from a P/T net equivalent of a use case obtained from the application of Mapping Rules 1 to 11. Procedure **GeneratePTNet** a formulation of these Mapping Rules, returns a P/T net from a use case. We assume relation **PlaceVertices** such that  $p$  being a place and  $v$  a vertex,  $p \times v \in \text{PlaceVertices}$  if vertex  $v$  corresponds to place

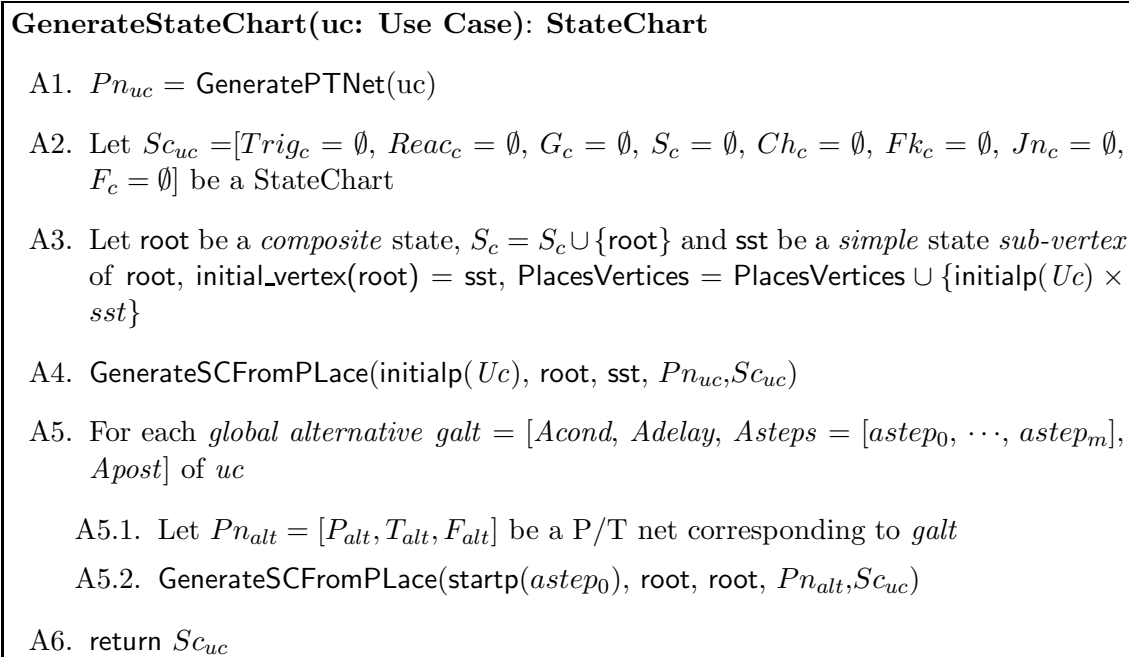


Figure 10: StateChart generation algorithm (Part 1).

*p*. In procedure **GenerateSCFromPlace**, we assume a relation **TransJoins** such that *t* being a transition and *j* a join pseudostate,  $t \times j \in \text{TransJoins}$  if *j* corresponds to *t*. Each use case corresponds to a *root* composite state that is supervertex of all other vertices. We consider *global alternatives* separately from the other alternatives and exploit StateCharts composite states properties. As an example, use case “Submit order” global alternative results in a transition from the root state *s0* to *s12*. This transition applies to all of *s0* sub-vertices. Procedure **GenerateSCFromPlace** considers a place and generates an edge in the format  $v_s \xrightarrow{[guards]trig/reacs} v_t$  by looking up *guards*, *triggers* and *reactions* from the place. Notice that use case P/T net properties are such that either all or none of the transitions from a place are decisions. Places from which all transitions are decisions correspond to choice pseudostates, while the other places correspond to simple states (C5). Concurrent sequences of places/transitions are mapped to *orthogonal states* (B2.1. and B2.2). Forking transitions correspond to fork pseudostates and join transitions to join pseudostates. As an example, Figure 14 shows a StateChart corresponding to use case *uc1* shown in Figure 6. We assume steps *1*, *1.2*, *2.2* are actor operations and steps *2*, *1.1*, *2.1*, *3* are system reactions. Use case *uc1* P/T net is shown in Figure 7.

## 5.2 Sequential integration of Use Cases

We use a variant of UML activity diagrams [21] called *StateChart-Charts*, to integrate sequentially related *main* use cases. A StateChart is not appropriate for use case integration as the properties discussed in Section 4.4 can not extend to use case model P/T nets without

**GenerateSCFromPlace(p: Place, supv: Vertex, v: Vertex, Pn = [P<sub>uc</sub>, T<sub>uc</sub>, F<sub>uc</sub>]: P/T net, S<sub>cuc</sub> = [Trig<sub>c</sub>, Time<sub>c</sub>, Reac<sub>c</sub>, G<sub>c</sub>, S<sub>c</sub>, Ch<sub>c</sub>, Fk<sub>c</sub>, Jn<sub>c</sub>, F<sub>c</sub>]: StateChart)**

B1. Mark  $p$  as visited

B2. For each transition  $t$  such that  $p \times t \in F_{uc}$ ,

B2.1. If  $|t \bullet| > 1$ , let  $so$  be an *orthogonal* state sub-vertex of  $supv$ , let  $sf$  be a *fork pseudostate* sub-vertex of  $supv$ , let  $cond$  be the condition corresponding to  $t$ ,  
 $F_c = F_c \cup \{v \times \{cond\} \times \emptyset \times \emptyset \times sf\}$

B2.1.1. For each  $t \times p_o \in F_{uc}$ , Let  $r$  be a *region* in  $so$  and state  $so_i$  the initial state of  $r$ ,  $F_c = F_c \cup \{sf \times \emptyset \times \emptyset \times \emptyset \times so_i\}$ ,  
GenerateSCFromPlace( $p_o, r, so_i, Pn, S_{cuc}$ )

B2.2. If  $|\bullet t| > 1$ ,

B2.2.1. If  $\exists t \times sj \in \text{TransJoins}$ , let  $sj$  be a *join pseudostate* sub-vertex of  $supv$ ,  
- add  $t \times sj$  to TransJoins  
- let  $t \times p_j \in F_{uc}$ ,  $s_j = \text{GatherReactions}(p_j, \text{Reacs}, \text{supv}, Pn, S_{cuc})$   
-  $F_c = F_c \cup \{sj \times \emptyset \times \emptyset \times \text{Reacs} \times s_j\}$

B2.2.2. Let  $t \times sj \in \text{TransJoins}$ ,  $F_c = F_c \cup \{v \times \emptyset \times \emptyset \times \emptyset \times sj\}$

B2.3. If  $|\bullet t| = |t \bullet| = 1$

B2.3.1. If  $t$  corresponds to *trig*, an *actor operation* or a *timeout*, let  $t \times p_j \in F_{uc}$   
-  $nv = \text{GatherReactions}(p_j, \text{Reacs}, \text{supv}, Pn, S_{cuc})$ ,  $F_c = F_c \cup \{v \times \emptyset \times \{trig\} \times \text{Reacs} \times nv\}$

B2.3.2. If  $t$  corresponds to *cond*, a *condition* let  $t \times p_j \in F_{uc}$   
-  $nv = \text{GatherReactions}(p_j, \text{Reacs}, \text{supv}, Pn, S_{cuc})$ ,  $F_c = F_c \cup \{v \times \{cond\} \times \emptyset \times \text{Reacs} \times nv\}$

B2.3.3. If  $t$  corresponds to *reac*, a *system operation*, let  $t \times p_j \in F_{uc}$   
-  $nv = \text{GatherReactions}(p_j, \text{Reacs}, \text{supv}, Pn, S_{cuc})$ ,  
-  $\text{Reacs} = \{reac\} \cup \text{Reacs}$ ,  $F_c = F_c \cup \{v \times \emptyset \times \emptyset \times \text{Reacs} \times nv\}$

Figure 11: StateChart generation algorithm (Part 2).



<p><b>GatherReactions(p: Place, Reacs: Set, supv: Vertex, Pn = [P<sub>uc</sub>, T<sub>uc</sub>, F<sub>uc</sub>]: P/T net, S<sub>cuc</sub> = [Trig<sub>c</sub>, Time<sub>c</sub>, Reac<sub>c</sub>, G<sub>c</sub>, S<sub>c</sub>, Ch<sub>c</sub>, Fk<sub>c</sub>, Jn<sub>c</sub>, F<sub>c</sub>]: StateChart): Vertex</b></p> <p>C.1. Let <math>t</math> be such that <math>p \times t \in F</math></p> <p>C.2. If <math>t</math> corresponds to the <i>null event</i>, let <math>t \times p_j \in F_{uc}</math>, <math>p = p_j</math>, GOTO C.1.</p> <p>C.3. If <math>t</math> corresponds to <i>reac</i>, a <i>system operation</i>, <math>Reacs = Reacs \cup \{reac\}</math>, let <math>t \times p_j \in F_{uc}</math>, <math>p = p_j</math>, GOTO C.1.</p> <p>C.4. If <math>p</math> is marked as visited, let <math>p \times sv \in PlacesVertices</math>, return <math>sv</math></p> <p>C.5. Let <math>sst</math> be:</p> <ul style="list-style-type: none"> <li>– a <i>choice</i> pseudostate <i>sub-vertex</i> of <math>supv</math>, if all transitions from <math>initialp</math> are <i>decisions</i>, or</li> <li>– a <i>simple state</i> <i>sub-vertex</i> of <math>supv</math>, otherwise</li> </ul> <p><math>PlacesVertices = PlacesVertices \cup \{p \times sst\}</math></p> <p>C.6. GenerateSCFromPLace(p, supv, sst, Pn<sub>uc</sub>, S<sub>cuc</sub>)</p> <p>C.7. return <math>sst</math></p>
--

Figure 12: StateChart generation algorithm (Part 3).

imposing unpractical constraints on use case sequencing. For instance, P/T nets *balance-ness* and hence *1-safety* would require every set of concurrent use cases to be subsequently synchronized. The “Online Broker System” example as presented in Section 3, would be in violation of such constraint. UML activity diagrams allows modeling of use case sequencing constraints with the formal interpretation provided by Mapping Rules 12 to 14.

### 5.2.1 StateChart-Charts

A StateChart-Chart is a subset of an activity diagram where *action nodes* are StateCharts. Formally a StateChart-Chart is a tuple  $[SChs, CNds, CFIs]$  with:  $SChs$  a set of StateCharts,  $CNds$  a set of Flow Nodes, and  $CFIs$  a StateChart-Chart flow relation. Elements in  $SChs$  are StateCharts generated from use cases. Flow Nodes include activity diagram *initial* nodes, *decision* nodes, *join* nodes, *fork* nodes and *merge* nodes [21]. The StateChart-Chart flow relation  $CFIs$  is a relation defined in domain  $(SEn_c \cup CNds)$  and range  $(SRoot_c \cup CNds)$  with  $SEn_c$  the union of all the StateCharts in  $SChs$  *use case enabling* vertices, and  $SRoot_c$  the union of all the StateCharts in  $SChs$  *root* nodes. A use case enabling vertex corresponds to a use case enabling directive in a StateChart. More precisely, given a step sequence  $step_0, \dots, step_k, step_l$  with  $step_l$  a use case enabling directive, the *target vertex* of the edge corresponding to step  $step_k$  in  $Schar$  is a use case enabling vertex for  $step_l$ . An element  $nd_s \times nd_a \in CFIs$  represents *control flow* from  $nd_s$  to  $nd_a$ . Control flows between nodes in

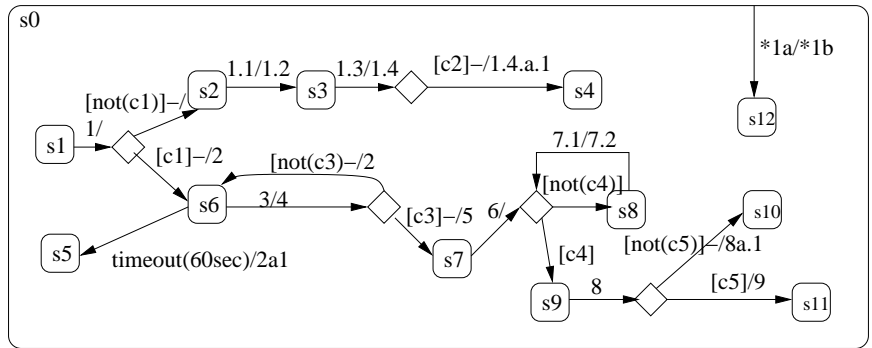


Figure 13: StateChart generated from use case “Submit order”. The event labels are the same as in Figure 5.

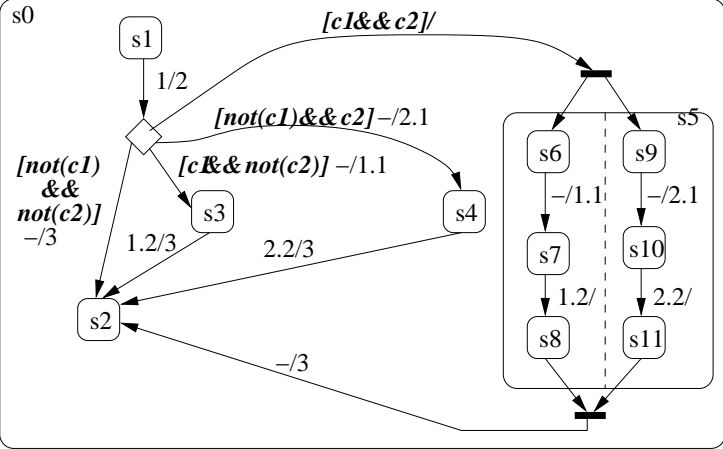


Figure 14: StateChart corresponding to use case *uc1* shown in Figure 6.

*CNDs* obey to UML activity diagram semantics. A flow from an enabling vertex captures the enabling of use cases by the use case from which the flow originates, while a flow ending at a use case root node signifies that the target use case is enabled and may start its execution. We consider that a new instance of activity (with a corresponding StateChart) is started for each execution of a use case.

Mappings between use cases sequencing constructs, P/T nets and StateChart-Charts are described in Figure 15. There is a straightforward mapping between use case sequencing concepts and UML activity diagram as we formally defined these sequencing concepts in term of P/T nets on one hand, and the semantics of UML activity diagrams are expressed in term of the Petri nets token game [21] on the other hand. States labelled *Se* in the StateChart-Chart fragments correspond to use case enabling directives. An enabling directive that refers to a single use case (with its matching *follow list*) corresponds to a simple flow from one use case to the other (Figure 15-a). Situations where a *follow list* refers to more than one use case correspond to a **join** when operator AND is used (Figure 15-b), or a **merge** when operator OR is used (Figure 15-c). Figures 15-d shows that a non-parallel use case enabling directive corresponds to a **decision** modeling a deferred choice based on the next event occurrence, while 15-e shows that a parallel use case enabling directive corresponds to a **fork**. Finally in a situation where there are steps after an enabling directive, Figure 15-f shows that in the corresponding StateChart-Chart, the steps after the directive are separated in a composite state. A fork node is used to model the concurrent execution of these steps with the enabled use cases.

### 5.2.2 Use Cases integration algorithm

Figure 16 shows an algorithm for StateChart-Charts generation. We assume the following functions and relations. Given a use case model  $\mathcal{M}$ ,  $\text{stateCharts}(\mathcal{M})$  is a set of StateCharts obtained from the *main* use cases in  $\mathcal{M}$  according to algorithm `GenerateStateChart`. Given a use case  $uc$ ,  $\text{stateChart}(uc)$  is a StateChart corresponding to  $uc$ . Function `enable_vertex` is such that `enable_vertex(edir, Schar)` is a use case enabling vertex corresponding to  $edir$  in a StateChart  $Schar$ . Given the set of all use case enabling vertices  $Ests$ , relation `nodes_ucases` is such that  $uc_o \times n_i \times uc_d \in \text{nodes\_ucases}$  if  $n_i \in (Ests \cup CNDs)$  is a node corresponding to the enabling of use case  $uc_d$  by use case  $uc_o$ . Function `enableNodes` is such that given  $(uc_i, uc_j)$  a pair of use cases,  $\text{enableNodes}(uc_i, uc_j) = \{rel \mid rel = uc_i \times n_{uci} \times uc_j \in \text{nodes\_ucases}\}$ . Function `unique_nodes_ucases` is similar to `nodes_ucases` except that every pair of use case corresponds to at most one node.

The StateChart-Charts algorithm is a formulation of Mapping Rules 12 to 14 based on the relation between P/T nets and StateChart-Charts depicted in Figure 15. Step 2.1 deals with the situation where an enabling directive is followed by further steps (Figure 15-f). Steps 2.2, 2.3 and 2.4 correspond to Mapping Rule 12. The parallel form of enabling directives are dealt with in step 2.2 (Figure 15-e), the non-parallel form in step 2.3 (Figure 15-d) and the situation where only one use case is enabled in step 2.4 (Figure 15-a). Step 3 of the algorithm corresponds to Mapping Rule 13, and consists of merging all enabling flows from an enabling use case targeted toward a use case. Mapping Rule 14 corresponds to step 4.

Use Case Construct	P/T net fragment	StateChart-Chart fragment
Title: uc0 1. ... ... n. enable: uc1 Title: uc1 ... Follows: uc0 (a)		 flow
Title: uc0 ... Follows: uc1 AND uc2 (b)		 join
Title: uc0 ... Follows: uc1 OR uc2 (c)	 merge	 merge
Title: uc0 1. ... ... n. enable: uc1, uc2 (d)	 decision	 decision
Title: uc0 1. ... ... n. enable in parallel: uc1, uc2 (e)	 fork	 fork
Title: uc0 1. ... ... m. enable ... m+1... ... n (f)		 fork

Figure 15: Mappings between use case sequencing constructs, P/T nets and StateChart-Charts.

**GenerateStateChartChart( $\mathcal{M} = [Act, Uc, Rel, InitialUc]$ : Use Case Model): StateChart-Chart**

1. Let  $Scc = [SChs = stateCharts(\mathcal{M}), CNds = \emptyset, CFls = \emptyset]$  be a StateChart-Chart.
2. For each  $uc \in M$ , let  $Sch_{uc}$  be  $stateChart(uc)$ .  
For each  $edir \in enable\_dirs(uc)$ , let  $s_e$  be  $enable\_vertex(edir, Sch_{uc})$ 
  - 2.1. If there are steps after  $edir$ , let  $Sch_n$  be the StateChart corresponding to the steps after  $edir$ ,  $CNds = CNds \cup \{En\}$  with  $En$  a *fork* node,  
-  $CFls = CFls \cup \{s_e \times En, En \times root(Sch_n)\}$ , let  $s_{en} = En$   
Else let  $s_{en} = s_e$
  - 2.2. If  $isParallel(edir)$ ,  $CNds = CNds \cup \{Nd_e\}$  with  $Nd_e$  a *fork* node,  
-  $CFls = CFls \cup \{s_{en} \times Nd_e\}$ ,  $\forall uc_i \in enabled\_uc(edir)$ ,  $nodes\_ucases = nodes\_ucases \cup \{uc \times Nd_e \times uc_i\}$
  - 2.3. If  $\neg isParallel$  and  $|enabled\_uc(edir)| > 1$   $CNds = CNds \cup \{Nd_e\}$  with  $Nd_e$  a *decision* node,  
-  $CFls = CFls \cup \{s_{en} \times Nd_e\}$ ,  $\forall uc_i \in enabled\_uc(edir)$ ,  $nodes\_ucases = nodes\_ucases \cup \{uc \times Nd_e \times uc_i\}$
  - 2.4. If  $|enabled\_uc(edir)| = 1$ , let  $enabled\_uc(edir) = \{uc_i\}$ ,  $nodes\_ucases = nodes\_ucases \cup \{uc \times Nd_e \times uc_i\}$
3. For each pair of *main use cases*  $(uc_i, uc_j)$ , let  $EnN$  be  $= enableNodes(uc_i, uc_j)$ , if  $|EnN| \geq 1$ , let  $Nm_u \in CNds$  be a *merge* node, for each  $uc_i \times n_{uci} \times uc_j \in EnN$ ,  $CFls = CFls \cup \{n_{uci} \times Nm_u\}$   $uc_i \times Nm_u \times uc_j \in unique\_nodes\_ucases$ .
4. For each *main use case*  $uc = [UCTitle, UCPrec, UCFoll, UCSt, UCAlt, UCPost]$ , let  $FolUC$  be  $followed\_ucases(UCFoll)$ ,
  - 4.1. If  $isSynchronized(UCFoll)$ , let  $Nf \in CNds$  be a *join* node,  $CFls = CFls \cup \{Nf \times root(uc)\}$ .
  - 4.2. If  $\neg isSynchronized(UCFoll)$  and  $|FolUC| > 1$ , let  $Nf \in CNds$  be a *merge* node,  $CFls = CFls \cup \{Nf \times root(uc)\}$ .
  - 4.3. If  $|FolUC| = 1$  let  $Nf = root(uc)$ .

For each  $n_{uci}$  such that  $\exists uc_i \in FolUC$  with  $uc_i \times n_{uci} \times uc \in unique\_places\_ucases$ ,  $CFls = CFls \cup \{n_{uci} \times Nf\}$
5.  $CNds = CNds \cup \{In\}$  with  $In$  an *initial* node. For each  $uc_i \in InitialUc$ ,  $CFls = CFls \cup \{In \times root(stateChart(uc_i))\}$
6. return  $Scc$

Figure 16: StateChart-Chart generation algorithm.

Step 4.2 handles synchronized follow lists (Figure 15-b), step 4.1 handles unsynchronized follow lists (Figure 15-c) and step 4.3 handles situations where only one use case is referred to in a follow list.

Figure 17 shows a StateChart-Chart corresponding to the use cases in Figure 8. A

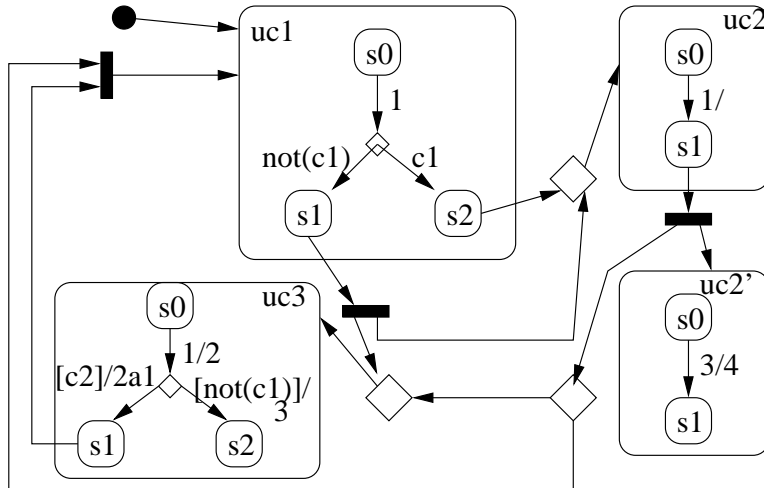


Figure 17: StateChart-Chart corresponding to use cases in Figure 8.

corresponding P/T net is presented in Figure 9. The example illustrates a situation where a use case is split by an enabling statement, with use case *uc2* corresponding to composite states *uc2* and *uc2'*. We assume use case *uc1* is the only initial use case among the three use cases.

### 5.3 Implementation and initial evaluation

The StateChart-Chart synthesis algorithm is implemented in a tool called Use Case Editor (UCed) [1]. The tool accepts use cases in the concrete syntax outlined in Section 3.2, checks for the consistency rules enumerated in Sections 3.2 and 4, and generates StateChart-Charts from consistent use cases. UCed provides a visualization mechanism for generated StateChart-Charts at two levels: the use case model level with StateCharts details hidden and use case StateCharts level. Figure 18 shows a StateChart-Chart generated from the “Online Broker System” example at the use case model level. Detailed StateChart for each use case can be seen by expanding use case nodes. Generated StateChart-Charts can be animated as prototypes. UCed includes a Simulator that provides a graphical user interface that allows “playing” generated StateChart-Charts. This gives an opportunity to validate use cases and their sequencing constraints.

We used UCed to carry different case studies aimed at validating our approach. The tool is being used to teach use case modeling in an academic setting. Moreover, UCed is released as an open source project for further feedback. The projects we experimented

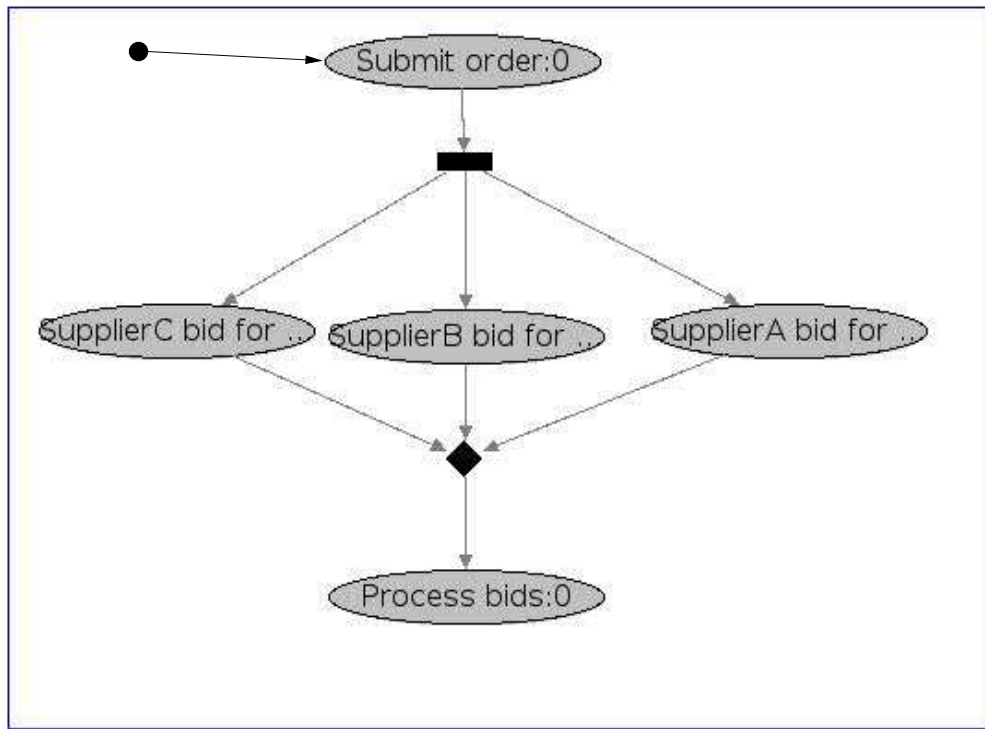


Figure 18: Use case model view of a StateChart-Chart generated from the “Online Broker System”.

with involve up to 20 use cases of varying degree of complexity. Because use cases are requirements artifacts, the number of use cases in project is typically limited. In any case, scaling to much larger projects should not be an issue because of the complexity of the state model construction algorithm. It is note-worthy that the complexity of StateChart and StateChart-Chart generation algorithms are both polynomial. Only Mapping Rule 11 induces an exponential complexity in term of the number of extension parts contributing to an extension point. However, this number is generally very limited in realistic examples.

## 6 Conclusions

This paper has proposed a formalization of textual use cases. We started from a formal definition of use case syntax; a UML metamodel as abstract syntax and a restricted natural language as concrete syntax. The main contribution of the paper is a definition of formal control-flow based semantics for use cases. We chose to express these semantics using the Basic Petri nets formalism. The choice of Basic Petri nets is sufficient for the assumed event model. Different assumptions or the consideration of control flow issues may ask for other forms of Petri nets such as Timed [30] or Coloreds Petri nets [12].

An equivalent P/T net may be constructed from a use case model granted that Consistency Rules specified in this paper are satisfied. These Consistency Rules should therefore be considered as part of a guideline for authoring use cases. We also developed algorithms for the synthesis of a two-level state model from a set of sequentially related use cases. These algorithms are implemented in a prototype tool and serve to validate the formal semantics.

We only consider UML use case *<< include >>* and *<< extend >>* relations in our formalization. We do not consider use cases *generalization* mainly because of uncertainties about the impact of this relation on textual use cases. We expect it would be possible to support use cases *generalization* in the future by extending use cases to Petri nets Mapping Rules. Our future works also includes system acceptance test generation from use cases based on the formal semantics.



## A Description of the Online Broker System use cases

<p>Title: SupplierA Bid System Under Design: Broker System Follows: Submit order Precondition: An Order has been broadcasted Follows Use Cases: Submit order Success Postcondition: SupplierA has submitted a bid STEPS</p> <ol style="list-style-type: none"><li>1. SupplierA receives the Order and examines it</li><li>2. SupplierA submits a Bid for the Order</li><li>3. The Broker System sends the Bid to the Customer</li><li>4. enable use case Process bids</li></ol> <p>ALTERNATIVES</p> <ol style="list-style-type: none"><li>1.a. SupplierA can not satisfy the Order<ol style="list-style-type: none"><li>1.a.1. SupplierA passes on the Order</li></ol></li></ol>
--

Figure 19: Description of use case “SupplierA Bid” in the Online Broker System.

Title: SupplierB Bid  
System Under Design: Broker System  
Follows: Submit order  
Precondition: An Order has been broadcasted  
Follows Use Cases: Submit order  
Success Postcondition: SupplierB has submitted a bid  
STEPS

1. SupplierB receives the Order and examines it
2. SupplierB submits a Bid for the Order
3. The Broker System sends the Bid to the Customer
4. enable use case Process bids

ALTERNATIVES

- 1.a. SupplierB can not satisfy the Order
  - 1.a.1. SupplierB passes on the Order

Figure 20: Description of use case “SupplierB” in the Online Broker System.

Title: SupplierC Bid  
System Under Design: Broker System  
Follows: Submit order  
Precondition: An Order has been broadcasted  
Follows Use Cases: Submit order  
Success Postcondition: SupplierC has submitted a bid  
STEPS

1. SupplierC receives the Order and examines it
2. SupplierC submits a Bid for the Order
3. The Broker System sends the Bid to the Customer
4. enable use case Process bids

ALTERNATIVES

- 1.a. SupplierC can not satisfy the Order
  - 1.a.1. SupplierC passes on the Order

Figure 21: Description of use case “SupplierC Bid” in the Online Broker System.

Title: Process Bids  
System Under Design: Broker System  
Follows Use Cases: SupplierA Bid OR SupplierB Bid OR SupplierC Bid  
Precondition: SupplierA has bid or SupplierB has bid or SupplierC has bid  
STEPS

1. Customer examines the bid
2. Customer signals the system to proceed with bid
3. include Handle Payment
4. System put an order with the selected bidder

Figure 22: Description of use case “Process bids” in the Online Broker System.

Title: Handle Payment  
System Under Design: Broker System STEPS

1. The Broker System asks the Customer for Credit Card information
2. The Customer provides her Credit Card information
3. The Broker System asks a Payment System to process the Customer’s Payment
4. The Broker System displays an acknowledgement message to the Customer

ALTERNATIVES

- 3.a. The Customer Payment is denied
  - 3.a.1. The Broker System displays a payment denied page

Figure 23: Description of use case “Process bids” in the Online Broker System.

## References

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