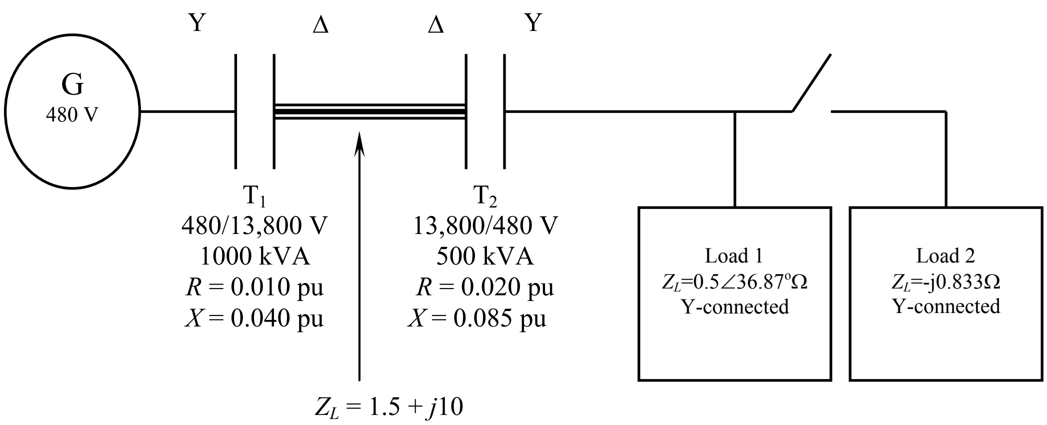
Problem 2-23

The following figure shows a power system consisting of a three-phase 480-V, 60-Hz generator supplying two loads through a transmission line with a pair of transformers at either end.

a. Sketch the per-phase equivalent circuit of this power system.
b. With the switch open, find the real power P, reactive power Q, and apparent power S supplied by the generator. What is the power factor of the generator?
c. With the switch closed, find the real power P reactive power Q, and apparent power S supplied by the generator. What is the power factor of the generator?
d. What are the transmission losses (transformer plus transmission line losses) in this system with the switch open? With the switch closed? What is the effect of adding load 2 to the system?



Region 1
S_{base1}=1000 kVA
V_{Lbase1}=480V

Region 2
S_{base2}=1000 kVA
$V_{L,\ base2}$= 13,800 V

Region 3
S_{base3}=1000 kVA
$V_{L,\ base3}$= 480 V

Solution: This problem can best be solved using the per-unit system of measurements. The power system can be divided into three regions by the two transformers. If the per-unit base quantities in Region 1 are chosen to be S_{base1} = 1000 kVA and $V_{L,base1}$ = 480 V, then the base quantities in Region 2 and 3 will be as shown above. The base impedance of each region will be:

$$Z_{base1} = \frac{3(V\varphi_1)^2}{S_{base1}} = \frac{3(277)^2}{1000\text{ kVA}} = 0.238\ \Omega$$

$$Z_{base2} = \frac{3(V\varphi_2)^2}{S_{base2}} = \frac{3(7967)^2}{1000\text{ kVA}} = 190.4\ \Omega$$

$$Z_{base3} = \frac{3(V\varphi_3)^2}{S_{base3}} = \frac{3(277)^2}{1000\text{ kVA}} = 0.238\ \Omega$$

a. To have the per-unit, per-phase equivalent circuit, we must convert each impedance in the system to per unit on the base of the region in which it is located. The impedance of transformer T_1 is already in per unit to the proper base

R_1, pu = 0.010
X_1, pu = 0.040

The impedance of transformer T_2 is already in per unit, but it is per-unit to the base of transformer T_2, so it must be converted to the base of the power system.

$$(R, X, Z)_{\text{pu on base 2}} = (R, X, Z)_{\text{pu on base 1}} \frac{(V_{base1})^2 (S_{base2})}{(V_{base2})^2 (S_{base1})}$$

$$R_{2,pu} = 0.02 \frac{(7967\ \text{V})^2 (1000\ \text{kVA})}{(7967)^2 (500\ \text{kVA})} = 0.040$$

$$X_{2,pu} = 0.085 \frac{(7967\ \text{V})^2 (1000\ \text{kVA})}{(7967)^2 (500\ \text{kVA})} = 0.170$$

The per unit impedance of the transmission line is

$$Z_{\text{line, pu}} = \frac{Z_{\text{line}}}{Z_{\text{base2}}} = \frac{1.5 + j10}{190.4} = 0.00788 + j0.0525$$

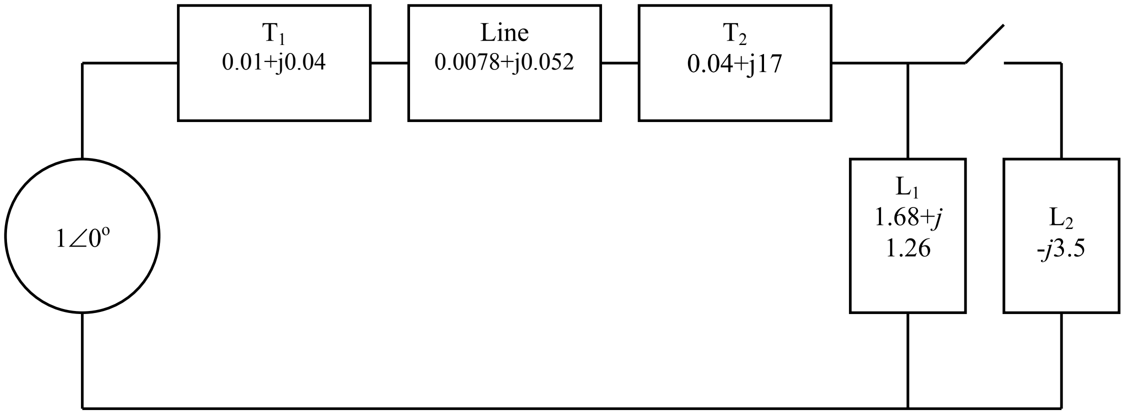
The per unit impedance of load 1 is

$$Z_{\text{load1,pu}} = \frac{Z_{\text{load1}}}{Z_{\text{base3}}} = \frac{0.5\angle 36.87^{\circ}}{0.238} = 1.681 + j1.261$$

The per unit impedance of load 2 is

$$Z_{\text{load2,pu}} = \frac{-j0.833}{0.238} = -j3.5$$

The per-unit, per-phase equivalent circuit is shown below



b. With the switch open, the equivalent impedance is

$$Z_T = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + 1.681 + j1.261$$
$$= 2.312\angle 41.2^o$$

The current is

$$I = \frac{1\angle 0^o}{2.312\angle 41.2^o} = 0.4325\angle -41.2^o \text{ A}$$

The load voltage is

$$V_{load,\,pu} = I\, Z_{load} = \left(0.4325\angle\text{-}41.2^o\right)\left(1.681 + j1.261\right) = 0.909\angle -4.3^o$$

The actual load voltage is 0.909×480=436 V.

The power supplied to the load is

$$P_{load,pu} = I^2\, R_{load} = \left(0.4325\right)^2\left(1.681\right) = 0.314$$
$$P_{load} == P_{load,pu}\, S_{base} = \left(0.314\right)\left(1000\text{ kVA}\right) = 314\text{ kW}$$

The power supplied by the generator

$$P_{G,pu} = VI\cos\theta = (1)(0.4325)\left(\cos 41.2^o\right) = 0.325$$
$$Q_{G,pu} = \text{VI}\sin\theta = (1)(0.4325)\left(\sin 41.2^o\right) = 0.285$$
$$S_{G,pu} = \text{VI} = (1)(0.4325) = 0.4325$$

Actual values are: P_G = 325 kW; Q_G = 285 kVAR; S_G = 432.5 kVA

The power factor is: cos 41.2° = 0.752 lagging

c. With the switch closed, the equivalent circuit is

$$Z_T = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + \frac{(1.681 + j1.26)(-j3.5)}{1.681 + j1.261 - j3.5} = 2.698\angle 5.6^o$$

The current is

$$I = \frac{1\angle 0^o}{2.698\angle 5.6^o} = 0.371\angle -5.6^o$$

The load voltage is

$$V_{load,pu} = I\,Z_{load} = \left(0.371\angle\text{-}5.6^o\right)(2.627 - j0.011) = 0.975\angle -5.6^o$$
$$Vload = (0.975)(480) = 468\text{ V}$$

The power supplied to the two loads is

$$P_{load,\ pu} = I^2 R_{load} = (0.371)2\,(2.672) = 0.361$$
$$P_{load} = (0.361)(1000\text{ kVA}) = 361\text{ kW}$$

The power supplied by the generator

$$P_{G,pu} = \text{VI}\cos\theta = (1)(0.371)\cos 5.6^o = 0.369$$
$$Q_{G,pu} == VI\sin\theta = (1)(0.371)\sin 5.6^o = 0.0369$$
$$S_{G,pu} = V\text{I} = 0.371$$

Multiply each value by 1000 kVA to get actual values: P_G = 369 kW; Q_G = 36 kVAR; S_G = 371 kVA. The power factor of the generator is cos 5.6° = 0.995 lagging.

d. The transmission losses with the switch open are:

$$P_{line,pu} = (0.4325)^2(0.00788) = 0.00147$$
$$P_{line} = (0.00147)(1000) = 1.47\text{ kW}$$

The transmission losses with the switch closed are

$$P_{line,pu} = (0.371)^2(0.00788) = 0.00108$$
$$P_{line} = (0.00108)(1000) = 1.08\text{ kW}$$

Load 2 improved the power factor of the system.