Workshop on Covering Arrays: Constructions, Applications and Generalizations

Plenary Talks

Rick Brewster, Thompson Rivers University
Graph Homomorphisms, an introduction
This talk is an introduction to the subject of graph homomorphisms. The concept of homomorphisms appears in many areas of mathematics, and the field of graph theory is no exception. However, until recently most graph theorists did not view graph homomorphisms as a central topic in the discipline. In their recent book "Graphs and homomorphisms", Hell and Nesetril make the case that "the time is ripe to introduce this exciting topic to a wider audience". In this talk we shall provide introductory concepts and examples, survey some history, and outline connections to other areas of mathematics and computer science. In particular, both categorical aspects and computational complexity will be examined.

Charles Colbourn, Arizona State University
Construction Techniques for Covering Arrays
The construction of covering arrays to minimize the number of test performed is a challenging problem. To date, computational methods have proved to be effective for "small" arrays; indeed for practical software tools, greedy methods are prevalent. However even the best of these do not appear to scale well to larger problems; either the size of the array appears to be unnecessarily large, or the time to produce the array appears prohibitive.

Consequently combinatorial and algebraic techniques have a substantial role to play, even in the construction of covering arrays of "moderate" size. In this talk we explore recursive constructions of covering arrays. We emphasize the cases with higher strength, since these are at present less amenable to algorithmic techniques.

We start with a 1987 construction by Roux, a simple cut-and-paste technique for binary arrays of strength three, juxtaposing two smaller arrays. We then generalize it to (1) more symbols, (2) more copies, and (3) higher strength. Then we turn to a powerful construction using perfect hash families and explore the relationships among perfect hash families, separating hash families, and covering arrays.

Throughout, numerous questions will be posed, and a few of them answered.

Peter Gibbons, University of Auckland
Computational Constructions of Combinatorial Structures
We survey popular methods for constructing and enumerating combinatorial structures. The techniques will be explained using examples involving various types of incidence structures, including triple systems, queens' domination, and covering arrays. Rather than purporting to be a research seminar, this is a relatively introductory tutorial aimed at those who wish to better understand computational methods in general and how to profitably apply them to their own structures of interest.

Alan Hartman, IBM Haifa Research Laboratory
Covering Arrays: Mathematical, Engineering, and Scientific Perspectives
There is no doubt that covering arrays are a fruitful and elegant area for mathematical research. Recently there have been some interesting developments in making these structures more easily available to the software engineering community. The mathematics and computer science research communities have made many claims for the usefulness of covering arrays in software engineering, and in particular in software and hardware testing. Engineers have recently called these results into question. We will discuss this debate and propose some scientific research activities which will help in settling these questions.

Brett Stevens, Carleton University
Covering Arrays and their Generalizations
I will briefly define covering arrays, outline their uses and anticipate the other talks in the workshop. I will then go on to describe several generalizations that have been made to the standard covering array model and assumptions. These have almost all been motivated by consideration of real world application circumstances. We will discuss optimization when we know of definite non-interaction of internal components; this uses graph homomorphism theory and is often called a covering array on a graph, G. We look at recent work on mixed factor levels and talk about mixed covering arrays on graphs. We also look at some work on uniform covering arrays, where each symbol occurs

nearly equally often in each factor. Another recent generalization is the use of a "don't care" symbol which has led to some beautiful optimizations of known constructions. Finally we look at covering arrays with forbidden pairs. We will show that, paradoxically, forbidding certain pairs of interactions can either reduce the size of an array or increase it often dramatically.

Doug R. Stinson, University of Waterloo
Tutorial on Orthogonal Arrays: Constructions, Bounds and Links to Error-correcting Codes
We begin by defining orthogonal arrays $OA(k, n)$ and briefly discussing their connection to sets of mutually orthogonal Latin squares. Next, we generalize the definition of OAs to higher lambda, and we discuss the Plackett-Burman bound and cases of equality. Then, we generalize the definition of OAs to higher strength and we discuss the links between linear OAs and linear codes. We then look briefly at nonlinear OAs and their relationship to nonlinear codes. Finally, we present a few results on OAs where the number of symbols is not a prime power. Throughout the talk, we are emphasizing constructions (sufficient conditions) and bounds (necessary conditions) for the objects under consideration. A few proofs are given along the way.

Contributed Talks

Robert F. Bailey, Queen Mary, University of London
Uncoverings-by-bases for Groups and Matroids
Let G be a permutation group acting on a finite set Ω. An *uncovering-by-bases* for G is a set $\mathcal{U}$ of bases for G (i.e. a sequence of points from Ω whose pointwise stabiliser is trivial) such that any r-subset of Ω is disjoint from at least one base in $\mathcal{U}$. These objects are closely related to covering designs, and they arise in the decoding algorithm for error-correcting codes which I describe in my DMD talk.

I will give some examples of some constructions, which use different techniques (such as finite fields and graph decompositions). Also, the definition generalises to matroid theory: I will also give a brief description of this, if time permits.

Frank E. Bennett, Mount Saint Vincent University
Perfect Mendelsohn Designs: A Brief Survey of Existence Results

Let v, k and λ be positive integers. A (v, k, λ)-*Mendelsohn design*, denoted briefly by (v, k, λ)-MD, is a pair $(X, \mathbf{B})$ where X is a v-set (of *points*) and $\mathbf{B}$ is a collection of cyclically ordered k-subsets of X (called *blocks*) such that every ordered pair of points of X are consecutive in exactly λ blocks of $\mathbf{B}$. If for all $t = 1, 2, \ldots, k-1$, every ordered pair of points of X are t-apart in exactly λ blocks of $\mathbf{B}$, then the (v, k, λ)-MD is called a *perfect* design and denoted briefly by (v, k, λ)-PMD. The basic necessary conditions for the existence of a (v, k, λ)-PMD are $v \geq k$ and $\lambda v(v-1) \equiv 0 \pmod{k}$. These conditions are known to be sufficient in most cases, but certainly not in all. For $k = 3, 4, 5, 6, 7$, very extensive investigations of (v, k, λ)-PMDs have now been carried out. In some of these cases, the results have been fairly conclusive. We shall provide a brief survey the known existence results. It is well known that an equivalent formulation for a set of $k-2$ mutually orthogonal Latin squares (MOLS) of order n is that of an *orthogonal array* OA$(n; k)$. This is an $n^2 \times k$ array whose entries come from an n-set X, and such that for any pair of columns every ordered pair of elements of X (not necessarily distinct) appear in the same row exactly once. It is also known that the existence of a $(v, k, 1)$-PMD implies the existence of an OA$(v; k)$, which is invariant under cyclic permutation of its columns.

Myra Cohen, University of Nebraska - Lincoln
Variable Strength Covering Arrays: Applications and Challenges
A covering array $CA(N; t, k, v)$ is an $N \times k$ array on v symbols such that for any $N \times t$ sub-array all ordered t-tuples occur at least once where t is called the strength of the array. A $VCA(N; t, k, v, C)$ is an $N \times k$ array such that any $N \times t$ sub-array contains all order t-tuples at least once, and where C defines a vector of m covering arrays, with $t\prime_1, t\prime_2, .., t\prime_m > t$ and where the columns in C form a subset of columns from k. In this work we discuss the use of variable strength covering arrays in practical applications for software testing and highlight some computational techniques for finding them. Little however, is knownabout direct mathematical constructions. We discuss the need for more research on this front and leave this as an open problem.

Lucia Gionfriddo, University of Catania
On the spectrum of Hexagon G-systems
Let G be a graph and let J be a family of subgraphs G' of G. A $G-system$ of order n and index ρ is a pair $\Sigma = (X, H)$,where X is a finite set of n

vertices and H is a collection ofedge disjoint graphs G (called blocks) which partitions the edgeset of complete graph ρK_n, with vertex set X. We say that aG-system Σ is $J-nesting$ if, for every fixed subgraph$G' \in J$, the collection of all the subgraphs G' contained inthe blocks of the $G-system$ Σ form a $G'-system$ with adue index λ. "Perfect hexagon triple systems" (studiedby S. Kucukcifci and C. Lindner (2004)) and "Perfect Dexagontriple systems" (studied by C. Lindener and A. Rosa (2006))can be considered following this definition. We have studied thespectrum for these J-nesting G-systems, determining it completely,in various cases: for the " Hexagon Quadrangle suystems", for the"Hexagon bi-quadrangle systems", for the "Hexagon kite systems".Other interesting cases can be considered.

Anant P. Godbole, East Tennessee State University

Partial Covering Arrays and a Generalized Erdos-Ko-Rado Property

The classical Erdős-Ko-Rado theorem states that if $k \leq \lfloor n/2 \rfloor$ then the largest family of pairwise intersecting k-subsets of $[n] = \{0, 1, \ldots, n\}$ is of size $\binom{n-1}{k-1}$. A family of k subsets satisfying this pairwise intersecting property is called an EKR family. We generalize the EKR property and provide asymptotic lower bounds on the size of the largest family $\mathcal{A}$ of k-subsets of $[n]$ that satisfies the following property: For each $A, B, C \in \mathcal{A}$, each of the four sets $A \cap B \cap C; A \cap B \cap C^C; A \cap B^C \cap C; A^C \cap B \cap C$ are non-empty. This generalized EKR (GEKR) property is motivated, generalizations are suggested, and a comparison is made with fixed weight 3-covering arrays. Our techniques are probabilistic, and reminiscent of those used in a paper of Godbole, Sunley and Skipper and in the work of Roux, as cited in the classic survey paper of Sloane.

Joint work with Particia A. Carey.

Roy Gourgi, *Covering designs - a new approach*

A new dynamic approach forfinding covering designs that minimizes resources (memory & cpu time) and a new formula that improves the lowest bounds.

Robert E. Jamison, Clemson University

Difference Sets, Bouchet Diagrams, and the Achromatic Index of Complete Graphs

The achromatic index $A(n)$ of the complete graph K_n is the *largest* number of colors that can be used to color the edges of K_n so that the following conditions are satisfied:

- edges which share a common endnode get different colors [proper], and
- given any two colors i and j there are edges e and f of those colors which do share a common endnode [completeness].

Although it is known that $A(n)$ grows asymptotically like $n^{3/2}$, very few exact values are known. In his study of $A(n)$, Andre Bouchet, introduced a technique for obtaining lower bounds. So far this technique remains largely unexplored, but it can be viewed as a relaxation of the notion of a difference set with a perfect matching. In this talk, I will describe efforts to produce Bouchet diagrams from odd difference sets and the structure which results.

Yu Lei, University of Texas at Arlington
Combinatorial Testing using Covering Arrays: Going beyond Pairwise Testing
A commonly used software testing technique is combinatorial testing, which involves constructing and using covering arrays so that every interaction between input parameters is exercised. Existing work on combinatorial testing has mainly focused on 2-way (pair-wise) interaction testing in which a covering array of strength two is used to exercise every interaction between any two parameters. In this talk, we will report on a project to develop new methods and tools for efficiently constructing covering arrays for up to 6-way interaction testing. A recent study of actual faults revealed that in certain software, faults may result from up to 6-way interactions. One of the main challenges is that the computational complexity of constructing covering arrays goes up rapidly as the degree of interaction increases. We will discuss the design and implementation of a tool called FireEye for multi-way testing. In particular, we will highlight several key data structures that are used to reduce the time and space requirements of the test generation process. The directions along which our work is being continued will also be discussed. This project is a collaboration effort between the US National Institute of Standards and Technology, George Mason University, and the University of Texas at Arlington.

(Joint work with Raghu Kacker)

Lucia Moura, University of Ottawa

Covering Arrays and Extremal Set-Partition Systems

In this talk, we will look at a covering array as a system of partitions of an n-set that presents non-empty intersection between any two parts of different partitions. We will look at how results for extremal set systems were used to settled the binary-alphabet case, and propose the study of set-partition systems in order to advance on larger alphabet cases. Some generalizations of the Erdos-Ko-Rado and Sperner's theorems for set-partition systems will be discussed (joint work with Meagher and Stevens). We will briefly look at questions involving higher strength.

Daniel Panario, Carleton University

Division of Trinomials by Pentanomials and Orthogonal Arrays

Consider a maximum-length binary shift-register sequence generated by a primitive polynomial f of degree m. Let C_n^f denote the set of all subintervals of this sequence with length n, where $m < n \leq 2m$, together with the zero vector of length n. Munemasa (1999) considered the case in which the polynomial f generating the sequence is a trinomial satisfying certain conditions. He proved that, in this case, C_n^f corresponds to an orthogonal array of strength 2 that has a property very close to being an orthogonal array of strength 3.

Munemasa's result was based on his proof that very few trinomials of degree at most $2m$ are divisible by the given trinomial f. We consider the case in which the sequence is generated by a pentanomial f satisfying certain conditions. Our main result is that no trinomial of degree at most $2m$ is divisible by the given pentanomial f, provided that f is not in a finite list of exceptions we give. As a corollary, we get that, in this case, C_n^f corresponds to an orthogonal array of strength 3.

(Joint work with Michael Dewar, Lucia Moura, Brett Stevens and Steven Wang)

George Sherwood, Testcover.com LLC

A Column Expansion Construction for Optimal and Near-Optimal Mixed Covering Arrays

This talk describes the construction of a strength 2 mixed covering array using an orthogonal array and one or more ordered designs. The constructed array may have several different alphabet sizes. In each step of the recursive expansion, the degree for a particular alphabet size is multiplied by the corresponding ordered design degree. The covering array size exceeds the optimal size by less than the order of the orthogonal array. Conditions for achieving optimal size are described.

Joe Yucas, Southern Illinois University
Some covering arrays of strength two
We present constructions of a few infinite families of covering arrays of strength two. These are special cases resulting from joint work with C. Colbourn, S. Martirosyan, G. Mullen, D. Shasha and G. Sherwood. With these constructions, the upper bounds on the sizes of many covering arrays are improved.

Latifa Zekaoui, University of Ottawa
Mixed Covering Arrays on Graphs
In this talk, we look at a generalization of covering arrays that considers both mixed alphabet sizes for different rows and a graph structure on the rows that prescribes the pair of rows for which we require the covering property. A (standard) covering array is a particular case where all alphabets are the same and G is the complete graph. We extend (to the mixed alphabet case) results by Meagher and Stevens (2005) for covering arrays on graphs related to graph homomorphisms. We give optimal constructions of mixed covering arrays for trees, cycles and bipartite graphs.

(Joint work with Karen Meagher and Lucia Moura)