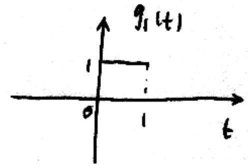
2002 Final

Question 1

$$g(t) = \frac{dx(t)}{d(t)} \Rightarrow$$



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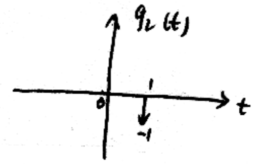


$$g(t) = g_1(t) + g_2(t)$$

$$G_1(jw) = \mathcal{F}[g_1(t)] = \frac{2\sin w\cdot\frac{1}{2}}{w} e^{-jw\frac{1}{2}}$$

$$G_2(jw) = \mathcal{F}[g_2(t)] = e^{-jw}$$

$$G(jw) = G_1(jw) + G_2(jw) = \frac{2\sin w\cdot\frac{1}{2}}{w} e^{-jw\frac{1}{2}} - e^{-jw}$$

$$X(jw) = \frac{G(jw)}{jw} + \pi G(0)\delta(w)$$

and $G(0) = 0$

so
$$X(jw) = \frac{\frac{2\sin w\frac{1}{2}}{w} e^{-jw\frac{1}{2}} - e^{-jw}}{jw}$$

$$= \frac{2e^{-j\frac{w}{2}}\sin\frac{w}{2}}{jw^2} - \frac{e^{-jw}}{jw}.$$

Question 2

(a) the impulse response of the system is

$$H(jw) = \frac{2}{(jw)^2 + 6jw + 8}$$

(b) the input $x(t) = e^{-2t} u(t)$

its Fourier transform is $X(jw) = \frac{1}{2+jw}$

so the output of this system is

$$Y(jw) = H(jw)\, X(jw)$$

$$= \frac{2}{(jw)^2 + 6jw + 8} \cdot \frac{1}{2+jw}$$

$$= \frac{2}{(2+jw)^2 (4+jw)}$$

$$= \frac{A_{11}}{2+jw} + \frac{A_{12}}{(2+jw)^2} + \frac{A_{13}}{4+jw}.$$

Based on the techniques of partial-fraction expansion

We obtained $A_{11} = -\frac{1}{2}$, $A_{12} = 1$, $A_{13} = \frac{1}{2}$

$$Y(jw) = \frac{-\frac{1}{2}}{2+jw} + \frac{1}{(2+jw)^2} + \frac{\frac{1}{2}}{4+jw}$$

the output of the system is then can be found by inverse transform

$$y(t) = \left(-\frac{1}{2}e^{-2t} + te^{-2t} + \frac{1}{2}e^{-4t}\right) u(t)$$

Question 3

$$X(jw) = \frac{1}{1-\frac{1}{2}e^{-jw}}$$

$$Y(jw) = \frac{1}{4}\cdot\frac{1}{1-\frac{1}{2}e^{-jw}} + \frac{1}{1-\frac{1}{4}e^{-jw}}$$

$$H(jw) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\frac{1}{4}\cdot\frac{1}{1-\frac{1}{2}e^{-jw}} + \frac{1}{1-\frac{1}{4}e^{-jw}}}{\frac{1}{1-\frac{1}{2}e^{-jw}}}$$

$$= \frac{1}{4} + \frac{1-\frac{1}{2}e^{-jw}}{1-\frac{1}{4}e^{-jw}} = \frac{1}{4} + 1 - \frac{\frac{1}{4}e^{-jw}}{1-\frac{1}{4}e^{-jw}}$$

$$= \frac{5}{4} - \frac{1}{4}\cdot\frac{1}{1-\frac{1}{4}e^{-jw}}\cdot e^{-jw}$$

$$h(n) = \mathcal{F}^{-1}[H(jw)] = \frac{5}{4}\delta(n) - \frac{1}{4}\left(\frac{1}{4}\right)^{n-1}u(n-1)$$

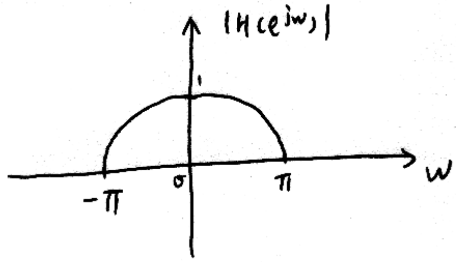
Question 4

(a) $H(e^{jw}) = \frac{1}{2} + \frac{1}{2}e^{-jw}$

$h(n) = \frac{1}{2}\delta(n) + \frac{1}{2}\delta(n-1)$

(b) $|H(e^{jw})| = \frac{1}{2}|1+e^{-jw}| = |\cos\frac{w}{2}|$

(c)



(d) low-pass filter

Question 5

(a) the differential expression for this system is

$$RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t) = x(t)$$

(b)

$$H(jw) = \frac{1}{LC(jw)^2 + RC(jw) + 1}$$

(c)

$L = 10 \times 10^{-3}\ H$

$C = 100 \times 10^{-6}\ F$

$R = 1\ \Omega$

$$H(jw) = \frac{\frac{1}{LC}}{(jw)^2 + \frac{R}{L}jw + \frac{1}{LC}} = \frac{\frac{1}{LC}}{(jw)^2 + 2\frac{1}{\sqrt{LC}} \cdot \frac{\sqrt{LC} \cdot R}{2L}jw + \frac{1}{LC}}$$

$$\zeta = \frac{1}{2}\sqrt{\frac{C}{L}}R = \frac{1}{2}\sqrt{\frac{100 \times 10^{-6}}{10 \times 10^{-3}}} < 1$$

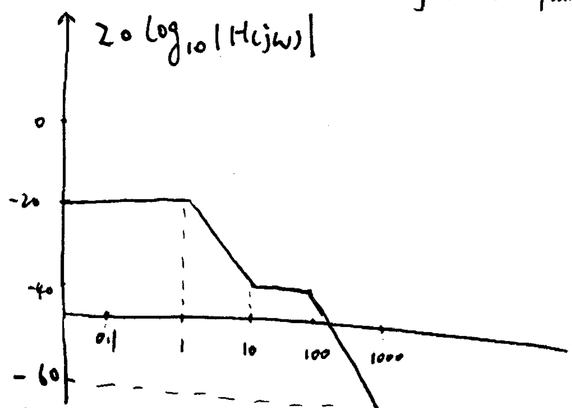
So the system is ~~over~~ - damped
under

Question 6

$$H(jw) = \frac{10+jw}{(1+jw)(100+jw)}$$

$$= \frac{1}{10} \cdot \frac{1}{1+jw} \cdot \frac{1}{1+\frac{jw}{100}} \cdot \left(1+\frac{jw}{10}\right)$$

The Bode plot for system magnitude function is below



The Bode plot for system phase function is below

Question 7

(a) $x(t) = 1 + \cos(100\pi t) + \cos(200\pi t)$

$f_M = 100$

So Nyquist rate $f_s = 2 f_M = 200$

(b) $y(t) = x^2(t) = (1 + \cos 100\pi t + \cos 200\pi t)^2$

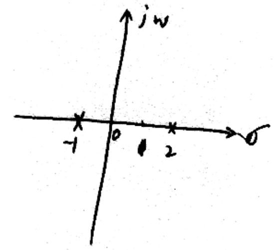
$f_M = 200$

So Nyquist rate $f_s = 2 f_M = 400.$

Question 8

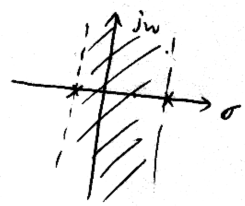
(a) $H(s) = \dfrac{1}{s^2 - s - 2}$

(b) $H(s) = \dfrac{1}{(s-2)(s+1)} \Rightarrow$



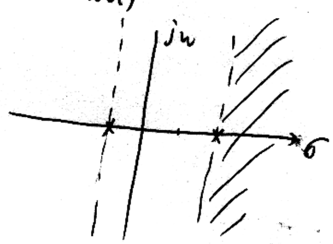
(c) $H(s) = \dfrac{A}{s-2} + \dfrac{B}{s+1} = \dfrac{\frac{1}{3}}{s-2} - \dfrac{\frac{1}{3}}{s+1}$

(i) The system is stable $\rightarrow$



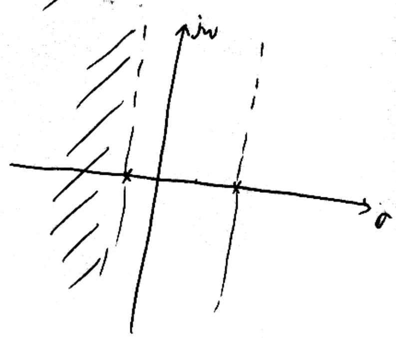
$h(t) = -\frac{1}{3} e^{2t} u(-t) - \frac{1}{3} e^{-t} u(t)$

(ii) The system is causal $\rightarrow$



$h(t) = (\frac{1}{3} e^{2t} - \frac{1}{3} e^{-t}) u(t)$

(iii) The system is neither stable nor causal



$h(t) = (-\frac{1}{3} e^{2t} + \frac{1}{3} e^{-t}) u(-t)$