9.21. (b)

$$e^{-4t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+4}, \qquad \mathcal{R}e\{s\} > -4.$$

Also,

$$e^{-5t}e^{j5t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+5-j5}, \qquad \mathcal{R}e\{s\} > -5.$$

and

$$e^{-5t}e^{-j5t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+5+j5}, \qquad \mathcal{R}e\{s\} > -5.$$

From this we obtain

$$e^{-5t}\sin(5t)u(t) = \frac{1}{2j}[e^{-5t}e^{j5t} - e^{-5t}e^{-j5t}]u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{5}{(s+5)^2+25},$$

where $\mathcal{R}e\{s\} > -5$. Therefore,

$$e^{-4t}u(t) + e^{-5t}\sin(5t)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s^2+15s+70}{s^3+14s^2+90s+100}, \qquad \mathcal{R}e\{s\} > -5.$$

(d)

$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \qquad \mathcal{R}e\{s\} > -2.$$

Using an approach along the lines of part (c), we obtain

$$e^{2t}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-2}, \qquad \mathcal{R}e\{s\} < 2.$$

From these we obtain

$$e^{-2|t|} = e^{-2t}u(t) + e^{2t}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{2s}{s^2-4}, \qquad -2 < \mathcal{R}e\{s\} < 2.$$

Using the differentiation in the s-domain property, we obtain

$$te^{-2|t|} \overset{\mathcal{L}}{\longleftrightarrow} -\frac{d}{ds}\left[\frac{2s}{s^2-4}\right] = -\frac{2s^2+8}{(s^2-4)^2}, \qquad -2 < \mathcal{R}e\{s\} < 2.$$

9.22. (a) From Table 9.2, we have

$$x(t) = \frac{1}{3}\sin(3t)u(t).$$

(b) From Table 9.2 we know that

$$\cos(3t)u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{s}{s^2+9}, \qquad \mathcal{R}e\{s\} > 0.$$

Using the time scaling property, we obtain

$$\cos(3t)u(-t) \stackrel{\mathcal{L}}{\longleftrightarrow} -\frac{s}{s^2+9}, \qquad \mathcal{R}e\{s\} < 0.$$

Therefore, the inverse Laplace transform of $X(s)$ is

$$x(t) = -\cos(3t)u(-t).$$

(c) From Table 9.2 we know that

$$e^t\cos(3t)u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{s-1}{(s-1)^2+9}, \qquad \mathcal{R}e\{s\} > 1.$$

Using the time scaling property, we obtain

$$e^{-t}\cos(3t)u(-t) \stackrel{\mathcal{L}}{\longleftrightarrow} -\frac{s+1}{(s+1)^2+9}, \qquad \mathcal{R}e\{s\} < -1.$$

Therefore, the inverse Laplace transform of $X(s)$ is

$$x(t) = -e^{-t}\cos(3t)u(-t).$$

(d) Using partial fraction expansion on $X(s)$, we obtain

$$X(s) = \frac{2}{s+4} - \frac{1}{s+3}.$$

From the given ROC, we know that $x(t)$ must be a two-sided signal. Therefore,

$$x(t) = 2e^{-4t}u(t) + e^{-3t}u(-t).$$

(g) We may rewrite $X(s)$ as

$$X(s) = 1 - \frac{3s}{(s+1)^2}.$$

From Table 9.2, we know that

$$tu(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s^2}, \qquad \mathcal{R}e\{s\} > 0.$$

Using the shifting property, we obtain

$$e^{-t}tu(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{(s+1)^2}, \qquad \mathcal{R}e\{s\} > -1.$$

Using the differentiation property,

$$\frac{d}{dt}[e^{-t}tu(t)] = e^{-t}u(t) - te^{-t}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{s}{(s+1)^2}, \qquad \mathcal{R}e\{s\} > -1.$$

Therefore,

$$x(t) = \delta(t) - 3e^{-t}u(t) - 3te^{-t}u(t).$$

9.28. **(a)** The possible ROCs are

(i) $\mathcal{R}e\{s\} < -2$.

(ii) $-2 < \mathcal{R}e\{s\} < -1$.

(iii) $-1 < \mathcal{R}e\{s\} < 1$.

(iv) $\mathcal{R}e\{s\} > 1$.

(b) (i) Unstable and anticausal.

(ii) Unstable and non causal.

(iii) Stable and non causal.

(iv) Unstable and causal.

9.31. **(a)** Taking the Laplace transform of both sides of the given differential equation and simplifying, we obtain

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}.$$

The pole-zero plot for $H(s)$ is as shown in Figure S9.31.

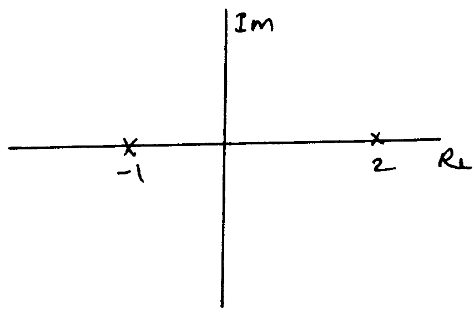


Figure S9.31

(b) The partial fraction expansion of $H(s)$ is

$$H(s) = \frac{1/3}{s-2} - \frac{1/3}{s+1}.$$

(i) If the system is stable, the ROC for $H(s)$ has to be $-1 < \mathcal{R}e\{s\} < 2$. Therefore,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t).$$

(ii) If the system is causal, the ROC for $H(s)$ has to be $\mathcal{R}e\{s\} > 2$. Therefore,

$$h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t).$$

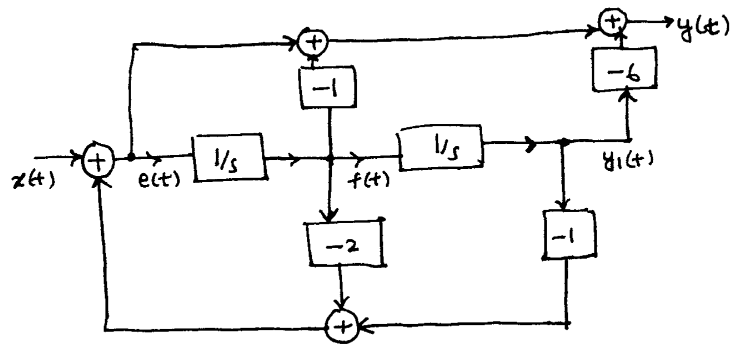
(iii) If the system is neither stable nor causal, the ROC for $H(s)$ has to be $\mathcal{R}e\{s\} < -1$. Therefore,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t).$$

9.35. **(a)** We may redraw the given block diagram as shown in Figure S9.35.

From the figure, it is clear that

$$\frac{F(s)}{s} = Y_1(s).$$



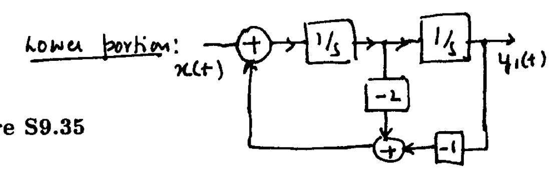


Figure S9.35

Therefore, $f(t) = dy_1(t)/dt$. Similarly, $e(t) = df(t)/dt$. Therefore, $e(t) = d^2y_1(t)/dt^2$. From the block diagram it is clear that

$$y(t) = e(t) - f(t) - 6y_1(t) = \frac{d^2y_1(t)}{dt^2} - \frac{dy_1(t)}{dt} - 6y_1(t).$$

Therefore,

$$Y(s) = s^2Y_1(s) - sY_1(s) - 6Y_1(s). \qquad \text{(S9.35-1)}$$

Now, let us determine the relationship between $y_1(t)$ and $x(t)$. This may be done by concentrating on the lower half of the above figure. We redraw this in Figure S9.35.

From Example 9.30, it is clear that $y_1(t)$ and $x(t)$ must be related by the following differential equation:

$$\frac{d^2y_1(t)}{dt^2} + 2\frac{dy_1(t)}{dt} + y_1(t) = x(t).$$

Therefore,

$$Y_1(s) = \frac{X(s)}{s^2 + 2s + 1}.$$

Using this in conjunction with eq (S9.35-1), we get

$$Y(s) = \frac{s^2 - s - 6}{s^2 + 2s + 1}X(s).$$

Taking the inverse Laplace transform, we obtain

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{d^2x(t)}{dt^2} - \frac{dx(t)}{dt} - 6x(t).$$

(b) The two poles of the system are at -1. Since the system is causal, the ROC must be to the right of $s = -1$. Therefore, the ROC must include the $j\omega$-axis. Hence, the system is stable.