7.3

(a) $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$

We can easily see that $X(j\omega) = 0$ for $|\omega| > 4000\pi$.

Therefore the Nyquist rate for the signal is

$\omega_N = 2 \times 4000\pi = 8000\pi$

(b) $x(t) = \dfrac{\sin(4000\pi t)}{\pi t}$

From Table 4.2 we know that, $X(j\omega)$ is a rectangular pulse for which $X(j\omega) = 0$ for $|\omega| > 4000\pi$

Therefore, the Nyquist rate for this signal is $\omega_N = 2 \times 4000\pi$
$= 8000\pi$

$$\left(\frac{\sin Wt}{\pi t} \longleftrightarrow X(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases} \right.$$

(c) $x(t) = \left(\dfrac{\sin(4000\pi t)}{\pi t}\right)^2$

$$\left(\frac{\sin 4000\pi t}{\pi t}\right)^2 \xleftrightarrow{F} \frac{1}{2\pi} \cdot F(j\omega) * F(j\omega)$$

where $F(j\omega) = F\left[\dfrac{\sin 4000\pi t}{\pi t}\right]$ is a rectangular pulse for which $(j\omega) = 0$ for $|\omega| > 4000\pi$

Therefore, $X(j\omega) = 0$ for $|\omega| > 8000\pi$, and the Nyquist rate is $\omega_N = 2 \times 8000\pi = 16000\pi$

7.4 $y(t) = x^2(t)$

(c) $x^2(t) \xleftrightarrow{F} \frac{1}{2\pi} X(j\omega) * X(j\omega) = Y(j\omega)$

We can guarantee that $Y(j\omega) = 0$ for $|\omega| > \omega_0$.

therefore the Nyquist rate for $y(t)$ is $2\omega_0$.

(d) $y(t) = x(t)\cos(\omega_0 t) \xleftrightarrow{FT} Y(j\omega) = (\frac{1}{2})\, X(j(\omega-\omega_0)) + (\frac{1}{2})\, X(j(\omega+\omega_0))$

We can guarantee that $Y(j\omega) = 0$ for $|\omega| > \omega_0 + \frac{\omega_0}{2}$. therefor, the Nyquist rate for $y(t)$ is $3\omega_0$.

7.6 Considering signal $w(t) = x_1(t)\, x_2(t)$. The Fourier Transform of $w(t)$ is given by

$$W(j\omega) = \frac{1}{2\pi}[X_1(j\omega) * X_2(j\omega)]$$

Since $X_1(j\omega) = 0$ for $|\omega| \geq \omega_1$ and $X_2(j\omega) = 0$ for $|\omega| \geq \omega_2$

We may conclude that

$W(j\omega) = 0$ for $|\omega| \geq \omega_1 + \omega_2$.

The Nyquist rate is $\omega_N = 2(\omega_1 + \omega_2)$, therefore, the minimum sampling period which would still allow $w(t)$ to be recovered is $T = \frac{2\pi}{\omega_s} = \pi / (\omega_1 + \omega_2)$.

7.22. Using the properties of Fourier Transform

we obtain $Y(j\omega) = X_1(j\omega) X_2(j\omega)$

Therefore $Y(j\omega) = 0$ for $|\omega| > 1000\pi$.

The Nyquist rate is $\omega_N = 2 \times 1000\pi = 2000\pi$.

The samping period T can at most be

$2\pi / (2000\pi) = 10^{-3}$ s.

Therefore we have to use $T < 10^{-3}$ sec in order to

be able to recover $y(t)$ from $y_p(t)$.

7.30. (a) Since $x_c(t) = \delta(t)$, we have

$$\frac{d y_c(t)}{dt} + y_c(t) = \delta(t)$$

Taking the Fourier Tranform we obtain

$$j\omega Y(j\omega) + Y(j\omega) = 1$$

$\therefore Y(j\omega) = \frac{1}{j\omega + 1}$ and $y_c(t) = e^{-t} u(t)$.

(b). Since $y_c(t) = e^{-t} u(t)$, $y[n] = y_c[nT] = e^{-nT} u[n]$

Therefore $Y(e^{j\omega}) = \frac{1}{1 - e^{-T} e^{-j\omega}}$

Also, $H(e^{j\omega}) = \dfrac{W(e^{j\omega})}{Y(e^{j\omega})} = \dfrac{1}{1/1-e^{-T}e^{-j\omega}}$

$$= 1 - e^{-T}e^{-j\omega}$$

Therefore $h[n] = \delta[n] - e^{T}\,\delta[n-1]$.