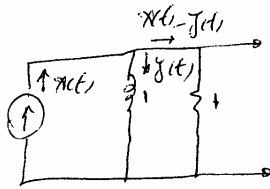


3.19

1/



a)

$$1. (x(t) - y(t)) = \frac{dy(t)}{dt}$$

$$\Rightarrow \frac{dy(t)}{dt} + y(t) = x(t)$$

$$b) \mathcal{F}\left\{\frac{dy(t)}{dt} + y(t)\right\} = \mathcal{F}\{x(t)\}$$

$$\Rightarrow (j\omega)y(j\omega) + y(j\omega) = x(j\omega) \Rightarrow [j\omega + 1]y(j\omega) = x(j\omega) \cdot \frac{1}{x(j\omega)} \Rightarrow$$

$$[1 + (j\omega)] \frac{y(j\omega)}{x(j\omega)} = 1 \Rightarrow H(j\omega) = \frac{1}{1 + (j\omega)}$$

$$c) x(t) = \cos t \xrightarrow[\text{Page 329}]{\text{Table 4.2}} \pi [\delta(\omega - 1) + \delta(\omega + 1)]$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) = \frac{\pi}{1 + (j\omega)} \{ \delta(\omega - 1) + \delta(\omega + 1) \} = \frac{\pi \delta(\omega - 1)}{1 + (j\omega)} + \frac{\pi \delta(\omega + 1)}{1 + j\omega}$$

$$= \frac{\pi}{1 + j} \delta(\omega - 1) + \frac{\pi}{1 - j} \delta(\omega + 1) = \frac{2\pi \delta(\omega - 1)}{2(1 + j)} + \frac{2\pi \delta(\omega + 1)}{2(1 - j)}$$

referring to table 4.2

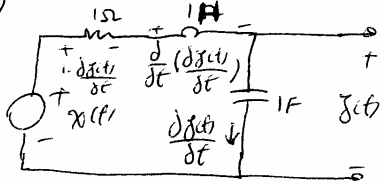
$$e^{j\omega t} \longleftrightarrow 2\pi \delta(\omega - 1)$$

$$e^{-j\omega t} \longleftrightarrow 2\pi \delta(\omega + 1)$$

$$y(t) = \frac{1}{2(1+j)} e^{jt} + \frac{1}{2(1-j)} e^{-jt} = \frac{e^{jt} - j e^{jt} - j e^{-jt} + e^{-jt}}{2(1-j^2)} = \frac{(e^{jt} + e^{-jt}) - j(e^{jt} - e^{-jt})}{2 \cdot 2}$$

$$= \frac{2 \cos t - 2j \sin t}{2 \cdot 2} = \frac{2}{2} \cdot \frac{\sin t + \cos t}{2} = \frac{1}{\sqrt{2}} \cos(t - \frac{\pi}{4})$$

3.20)



$$a) \quad x(t) = \frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t)$$

$$b) \quad \mathcal{F} \left\{ \frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) \right\} = \mathcal{F} \{ x(t) \}$$

$$\Rightarrow (j\omega)^2 Y(j\omega) + (j\omega) Y(j\omega) + Y(j\omega) = X(j\omega) \Rightarrow [(j\omega)^2 + (j\omega) + 1] Y(j\omega) = X(j\omega)$$

$$\Rightarrow [(j\omega)^2 + (j\omega) + 1] \frac{Y(j\omega)}{X(j\omega)} = 1 \Rightarrow H(j\omega) = \frac{1}{(j\omega)^2 + (j\omega) + 1}$$

$$c) \quad x(t) = \sin t \xleftrightarrow{\text{table 4.2}} \frac{\pi}{j} [\delta(\omega-1) - \delta(\omega+1)]$$

$$Y(j\omega) = H(j\omega) X(j\omega) = \frac{1}{(j\omega)^2 + (j\omega) + 1} \cdot \frac{\pi}{j} [\delta(\omega-1) - \delta(\omega+1)] =$$

$$\frac{\pi}{j} \cdot \frac{1}{\underbrace{(j)^2 + (j) + 1}_{=1}} \delta(\omega-1) - \frac{\pi}{j} \delta(\omega+1) \cdot \frac{1}{\underbrace{(-j)^2 + (-j) + 1}_{=1}} =$$

$$\frac{\pi}{j} \cdot \frac{1}{j} \delta(\omega-1) - \frac{\pi}{j} \cdot \frac{1}{j} \delta(\omega+1) = -\pi [\delta(\omega-1) + \delta(\omega+1)]$$

$$\Rightarrow y(t) = -\cos t$$

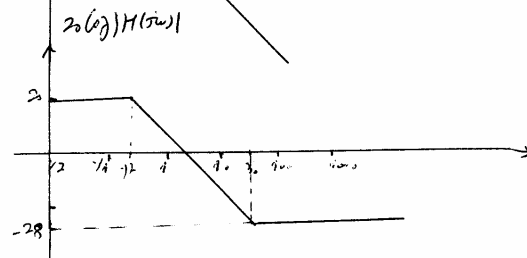
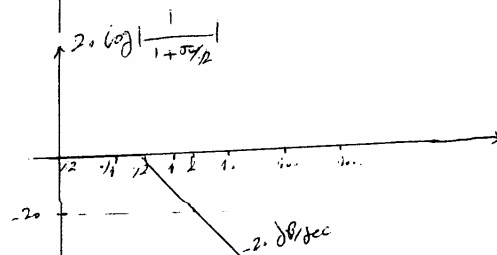
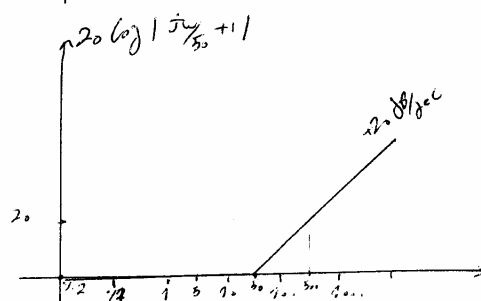
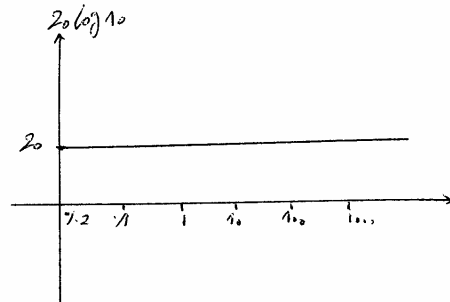
6.10) b)

3/

$$H(j\omega) = \frac{1}{12} \left(\frac{j\omega + 50}{j\omega + 12} \right) = \frac{1}{12} \cdot 50 \left(\frac{j\omega/50 + 1}{j\omega/12 + 1} \right) = 10 \left(\frac{j\omega/50 + 1}{j\omega/12 + 1} \right)$$

$$\Rightarrow 20 \log |H(j\omega)| = 20 \log 10 + 20 \log |j\omega/50 + 1| - 20 \log |j\omega/12 + 1|$$

$$= 20 + 20 \log |j\omega/50 + 1| + 20 \log \left| \frac{1}{1 + j\omega/12} \right|$$



6.11, a)

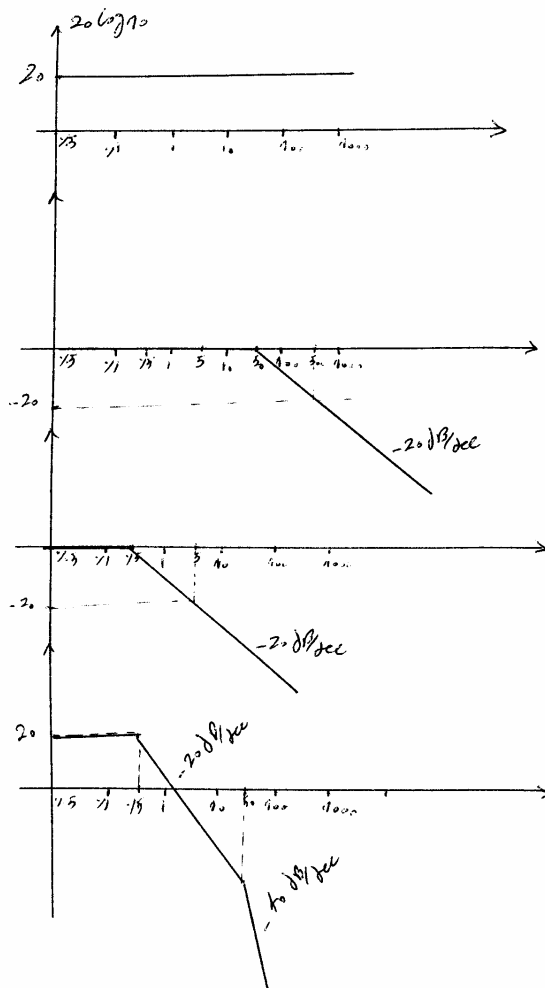
4

$$H(j\omega) = \frac{250}{(j\omega + 50)(j\omega + 15)} = \frac{250}{(j\omega + 50)(j\omega + 15)} = \frac{250}{50 \cdot 15} \cdot \frac{1}{(j\omega/50 + 1)(j\omega/15 + 1)}$$

$$= 10 \cdot \frac{1}{(j\omega/50 + 1)(j\omega/15 + 1)}$$

$$\Rightarrow 20 \log |H(j\omega)| = 20 \log 10 - 20 \log |j\omega/50 + 1| - 20 \log |j\omega/15 + 1| =$$

$$20 + 20 \log \left| \frac{1}{1 + j\omega/50} \right| + 20 \log \left| \frac{1}{1 + j\omega/15} \right|$$



6.15)

5

$$a) H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{(j\omega)^2 + 4(j\omega) + 4} = \frac{1}{4} \cdot \frac{1}{\left(\frac{j\omega}{2}\right)^2 + 2 \cdot \left(\frac{j\omega}{2}\right) + 1}$$

$$\Rightarrow \omega_n = 2 \quad 2\zeta\omega_n = 2 \Rightarrow \zeta = 1 \Rightarrow \text{critical damped}$$

$$b) H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{7}{5(j\omega)^2 + 4(j\omega) + 5} = \frac{7}{5} \cdot \frac{1}{(j\omega)^2 + \frac{4}{5}(j\omega) + 1}$$

$$\Rightarrow 2\zeta = \frac{4}{5} \rightarrow \zeta = \frac{2}{5} < 1 \Rightarrow \text{under damped}$$

$$c) H(j\omega) = \frac{1}{(j\omega)^2 + 20(j\omega) + 1}$$

$$\Rightarrow 2\zeta = 20 \rightarrow \zeta = 10 > 1 \Rightarrow \text{over damped}$$