5.1)

a)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} (\tfrac{1}{2})^{n-1} u[n-1] e^{-j\omega n} = \sum_{n=1}^{\infty} (\tfrac{1}{2})^{n-1} e^{-j\omega n}$$

let $n' = n-1$

$$\Rightarrow X(e^{j\omega}) = \sum_{n'=0}^{\infty} (\tfrac{1}{2})^{n'} e^{-j\omega(n'+1)} = \sum_{n'=0}^{\infty} (\tfrac{1}{2})^{n'} e^{-j\omega n'} \cdot e^{-j\omega} = e^{-j\omega} \sum_{n'=0}^{\infty} (\tfrac{1}{2} e^{-j\omega})^{n'}$$

$$= e^{-j\omega} \cdot \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

b)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} (\tfrac{1}{2})^{|n-1|} e^{-j\omega n} = \underbrace{\sum_{n=-\infty}^{0} (\tfrac{1}{2})^{-(n-1)} e^{-j\omega n}}_{X_1(e^{j\omega})} +$$

$$\underbrace{\sum_{n=+1}^{+\infty} (\tfrac{1}{2})^{n-1} e^{-j\omega n}}_{X_2(e^{j\omega})}$$

$$X_1(e^{j\omega}) = \sum_{n=-\infty}^{0} (\tfrac{1}{2})^{-(n-1)} e^{-j\omega n} = (\tfrac{1}{2}) \sum_{n'=0}^{\infty} (\tfrac{1}{2})^{n'} e^{-j\omega(-n')} = (\tfrac{1}{2}) \sum_{n'=0}^{\infty} (\tfrac{1}{2})^{n'} e^{j\omega n'}$$

let $n' = -n$

$$= (\tfrac{1}{2}) \sum_{n'=0}^{\infty} (\tfrac{1}{2} e^{j\omega})^{n'} = (\tfrac{1}{2}) \cdot \frac{1}{1 - \frac{1}{2} e^{j\omega}}$$

$$X_2(e^{j\omega}) = \sum_{n=1}^{\infty} (\tfrac{1}{2})^{n-1} e^{-j\omega n}$$

let

$n' = n-1$

$$\Rightarrow X_2(e^{j\omega}) = \sum_{n'=0}^{\infty} (\tfrac{1}{2})^{n'} e^{-j\omega(n'+1)} = e^{-j\omega} \sum_{n'=0}^{\infty} (\tfrac{1}{2}e^{-j\omega})^{n'} = e^{-j\omega} \cdot \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

therefore,

$$X(e^{j\omega}) = \cancel{\ } \cdot \frac{1/2}{1-\frac{1}{2}e^{j\omega}} + e^{-j\omega} \cdot \frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{0.75e^{-j\omega}}{1.25-\cos\omega}$$

5.6)

a)

$$x[1-n] = x[-(n-1)] \longleftrightarrow e^{-j\omega} X(e^{-j\omega})$$

$$x[-1-n] = x[-(n+1)] \longleftrightarrow e^{j\omega} X(e^{-j\omega})$$

$$\Rightarrow x[1-n] + x[-1-n] \longleftrightarrow 2\cos\omega \cdot X(e^{-j\omega}) = (e^{-j\omega} + e^{j\omega}) X(e^{-j\omega})$$

b)

$$x[-n] \longleftrightarrow X(e^{-j\omega})$$

$$x^*[-n] \longleftrightarrow X^*(e^{j\omega})$$

$$\Rightarrow \frac{x^*[-n] + x[n]}{2} = \frac{X^*(e^{j\omega}) + X(e^{j\omega})}{2} = \mathrm{Re}\{X(e^{j\omega})\}$$

5.13)

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{-12+5e^{-j\omega}}{12-7e^{-j\omega}+e^{-2j\omega}}$$

$$h_1[n] = (\tfrac{1}{3})^n u[n] \overset{DFT}{\longleftrightarrow} \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{-12+5e^{-j\omega}}{(e^{-j\omega}-4)(e^{-j\omega}-3)} = \frac{A}{(e^{-j\omega}-4)} + \frac{B}{e^{-j\omega}-3}$$

$$A = (e^{-j\omega}-4)H(e^{+j\omega})\Big|_{e^{-j\omega}=4} = \frac{-12+20}{1} = 8$$

$$B = (e^{-j\omega}-3)H(e^{j\omega})\Big|_{e^{-j\omega}=3} = \frac{-12+15}{3-4} = -3$$

$\Rightarrow$

$$H(e^{j\omega}) = \frac{8}{e^{-j\omega}-4} - \frac{3}{e^{-j\omega}-3} = \frac{-8/4}{1-\frac{e^{-j\omega}}{4}} - \frac{-3/3}{1-\frac{e^{-j\omega}}{3}} = \frac{-2}{1-\frac{e^{-j\omega}}{4}} + \frac{1}{1-\frac{e^{-j\omega}}{3}}$$

So,

$$H_2(e^{j\omega}) = H(e^{j\omega}) - H_1(e^{j\omega}) = \frac{-2}{1-\frac{e^{-j\omega}}{4}} + \frac{1}{1-\frac{e^{-j\omega}}{3}} - \frac{1}{1-\frac{1}{3}e^{-j\omega}} = \frac{-2}{1-\frac{e^{-j\omega}}{4}}$$

$$\Rightarrow h_2[n] = -2(\tfrac{1}{4})^n u[n]$$

5.29

a)

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

i)

$$X(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}}$$

therefore,

$$Y(e^{j\omega}) = \left[\frac{1}{1-\frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] = \frac{-2}{1-\frac{1}{2}e^{-j\omega}} + \frac{3}{1-\frac{3}{4}e^{-j\omega}}$$

$\Rightarrow$

$$y[n] = -2\left(\tfrac{1}{2}\right)^n u[n] + 3\left(\tfrac{3}{4}\right)^n u[n]$$

ii)

$$X(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}$$

therefore,

$$Y(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}\cdot\frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}} - \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}$$

$\Rightarrow$

$$y[n] = 4\left(\tfrac{1}{2}\right)^n u[n] - 2\left(\tfrac{1}{4}\right)^n u[n] - 1(n+1)\left(\tfrac{1}{4}\right)^n u[n].$$

iii)

$$X(e^{j\omega}) = 2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-(2k+1)\pi)$$

therefore

$$Y(e^{j\omega}) = \left[2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-(2k+1)\pi)\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] = \frac{4\pi}{3}\sum_{k=-\infty}^{\infty}\delta(\omega-(2k+1)\pi)$$

$$= \frac{2}{3}\left\{2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-(2k+1)\pi)\right.$$

taking the inverse Fourier transform, we obtain

$$x[n] = \frac{2}{3}(-1)^n$$

5.33

a)

$$\mathcal{F}\{y[n]+\tfrac{1}{2}y[n-1]\}=\mathcal{F}\{x[n]\}$$

$$\Rightarrow Y(e^{j\omega})+\tfrac{1}{2}e^{-j\omega}Y(e^{j\omega})=X(e^{j\omega})$$

$$\Rightarrow \left[1+\tfrac{1}{2}e^{-j\omega}\right]\frac{Y(e^{j\omega})}{X(e^{j\omega})}=1 \Rightarrow H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

b)

i)

$$X(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

therefore

$$Y(e^{j\omega})=\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{1+\frac{1}{2}e^{-j\omega}}\right]$$

$$=\frac{1/2}{1-\frac{1}{2}e^{-j\omega}}+\frac{1/2}{1+\frac{1}{2}e^{-j\omega}}$$

$$\Rightarrow y[n]=\tfrac{1}{2}(\tfrac{1}{2})^n u[n]+\tfrac{1}{2}(-\tfrac{1}{2})^n u[n]$$

iii)

$$X(e^{j\omega})=1+\tfrac{1}{2}e^{-j\omega}$$

therefore

$$Y(e^{j\omega})=1$$

$$\Rightarrow y[n]=\delta[n]$$

5.33

c)

iii)

$$Y(e^{j\omega}) = \left[\frac{1}{(1+\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}\right]\cdot\left[\frac{1}{1+\frac{1}{2}e^{-j\omega}}\right]$$

$$= \frac{2/3}{(1+\frac{1}{2}e^{-j\omega})^2} + \frac{2/9}{1+\frac{1}{2}e^{-j\omega}} + \frac{1/9}{1-\frac{1}{4}e^{-j\omega}}$$

$$\Rightarrow y[n] = \frac{2}{3}(n+1)(-\frac{1}{2})^n u[n] + \frac{2}{9}(-\frac{1}{2})^n u[n] + \frac{1}{9}(\frac{1}{4})^n u[n]$$

iv)

$$Y(e^{j\omega}) = [1+2e^{-3j\omega}]\cdot\frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

$$= \frac{1}{1+\frac{1}{2}e^{-j\omega}} + \frac{2e^{-3j\omega}}{1+\frac{1}{2}e^{-j\omega}}$$

$$\Rightarrow y[n] = (-\frac{1}{2})^n u[n] + 2(-\frac{1}{2})^{n-3} u[n-3]$$