4.1 ①

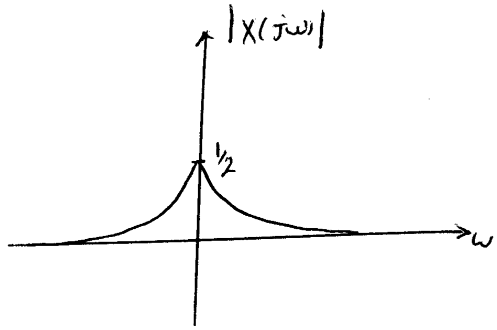
a) $x(t) = e^{-2(t-1)} u(t-1)$

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-2(t-1)} u(t-1)\, e^{-j\omega t}\, dt$$

$$= \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t}\, dt$$

$$= e^{-j\omega} / (2+j\omega)$$

$$|X(j\omega)| = \frac{|e^{-j\omega}|}{|2+j\omega|} = \frac{1}{\sqrt{4+\omega^2}}$$



b) $x(t) = e^{-2|t-1|}$

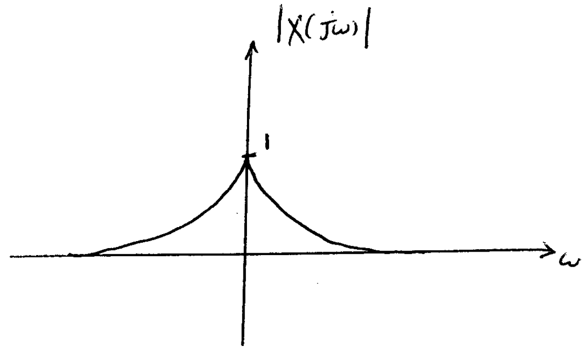
$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-2|t-1|} e^{-j\omega t}\, dt$$

$$= \int_{1}^{+\infty} e^{-2(t-1)} e^{-j\omega t}\, dt + \int_{-\infty}^{1} e^{2(t-1)} e^{-j\omega t}\, dt$$

$$= \frac{e^{-j\omega}}{2+j\omega} + \frac{e^{-j\omega}}{2-j\omega}$$

$$= \frac{4e^{-j\omega}}{4+\omega^2}$$

$$|X(j\omega)| = \frac{|4e^{-j\omega}|}{|4+\omega^2|} = \frac{4}{4+\omega^2}$$



4.6

a) $x(t) \xleftrightarrow{FT} X(j\omega)$

using the time reversal property $\left(x(-t) \xleftrightarrow{FT} X(-j\omega)\right)$, we have:

$$x(-t) \xleftrightarrow{FT} X(-j\omega)$$

using the time shifting property $\left(x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega)\right)$, we have

$$x(-t+1) \xleftrightarrow{FT} e^{-j\omega} X(-j\omega)$$

$$x(-t-1) \xleftrightarrow{FT} e^{j\omega} X(-j\omega)$$

therefore,

$$x_1(t) = x(-t+1) + x(-t-1) \xleftrightarrow{FT} e^{-j\omega} X(-j\omega) + e^{j\omega} X(-j\omega) = 2\cos\omega \cdot X(-j\omega)$$

b) using the time scaling property $\left(x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right) \right)$, we have:

$$x(3t) \overset{FT}{\longleftrightarrow} \frac{1}{3} X\left(j\frac{\omega}{3}\right)$$

using the time shifting property on this, we have:

$$x_2(t) = x(3(t-2)) \overset{FT}{\longleftrightarrow} e^{-2j\omega} \frac{1}{3} X\left(j\frac{\omega}{3}\right)$$

4.19)

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Since it is given that $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$, we can compute $Y(j\omega)$ to be

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

Since, $H(j\omega) = \frac{1}{3+j\omega}$, we have

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

Taking the inverse Fourier transform of $X(j\omega)$, we have

$$x(t) = e^{-4t}u(t).$$

4.23

b)

$$x_0(t) = \begin{cases} e^{-t} & 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\Rightarrow \quad X_0(j\omega) = \frac{1 - e^{-(1+j\omega)}}{1+j\omega}$$

$x_2(t) = x_0(t) - x_0(-t)$

using the linearity and time reversal properties of the Fourier transform we have:

$$X_2(j\omega) = X_0(j\omega) - X_0(-j\omega) = j\left[\frac{-2\omega + 2e^{-1}\sin\omega + 2\omega e^{-1}\cos\omega}{1+\omega^2}\right]$$

c)

$x_3(t) = x_0(t) + x_0(t+1)$

using the linearity and time-shifting properties of the Fourier transform we have.

$$X_3(j\omega) = X_0(j\omega) + e^{j\omega} X(+j\omega) = \frac{1 + e^{j\omega} - e^{-1}(1+e^{-j\omega})}{1+j\omega}$$

4.26

i)

$$Y(j\omega) = X(j\omega)\,H(j\omega) = \left[\frac{1}{(2+j\omega)^2}\right]\left[\frac{1}{4+j\omega}\right]$$

$$= \frac{1/4}{4+j\omega} - \frac{1/4}{2+j\omega} + \frac{1/2}{(2+j\omega)^2}$$

Taking the inverse Fourier transform we obtain:

$$y(t) = \frac{1}{4}e^{-4t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t)$$

ii)

$$Y(j\omega) = X(j\omega)\,H(j\omega)$$

$$= \left[\frac{1}{1+j\omega}\right]\left[\frac{1}{1-j\omega}\right]$$

$$= \frac{1/2}{1+j\omega} + \frac{1/2}{1-j\omega}$$

Taking the inverse Fourier transform, we obtain

$$y(t) = \frac{1}{2}e^{-|t|}$$

4.33

a) Taking The fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{2}{-\omega^2+2j\omega+8}$$

using partial fraction expansion, we obtain.

$$H(j\omega)=\frac{1}{j\omega+2}-\frac{1}{j\omega+4}$$

Taking the inverse Fourier transform,

$$h(t)=e^{-2t}u(t)-e^{-4t}u(t)$$

b)

For the given signal $x(t)$, we have

$$X(j\omega)=\frac{1}{(2+j\omega)^2}$$

Therefore,

$$Y(j\omega)=X(j\omega)\,H(j\omega)=\frac{2}{(-\omega^2+2j\omega+8)\,(2+j\omega)^2}$$

using partial fraction expansion, we obtain

$$Y(j\omega)=\frac{1/4}{j\omega+2}-\frac{1/2}{(j\omega+2)^2}+\frac{1}{(j\omega+2)^3}-\frac{1/4}{j\omega+4}$$

Taking the inverse Fourier transform,

$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$$

c) Taking the fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2(-\omega^2-1)}{-\omega^2+\sqrt{2}j\omega+1}$$

Using partial fraction expansion, we obtain

$$H(j\omega) = 2 + \frac{-\sqrt{2}-2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}+j\sqrt{2}}{2}} + \frac{-\sqrt{2}+2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}-j\sqrt{2}}{2}}$$

Taking the inverse Fourier transform:

$$h(t) = 2\delta(t) - \sqrt{2}(1+2j)e^{-(1+j)t/\sqrt{2}}u(t) - \sqrt{2}(1-2j)e^{-(1-j)t/\sqrt{2}}u(t)$$