3.4. Since $\omega_0 = \pi$, $T = 2\pi/\omega_0 = 2$. Therefore,

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}dt$$

Now,

$$a_0 = \frac{1}{2}\int_0^1 1.5dt - \frac{1}{2}\int_1^2 1.5dt = 0$$

and for $k \neq 0$

$$\begin{aligned} a_k &= \frac{1}{2}\int_0^1 1.5e^{-jk\pi t}dt - \frac{1}{2}\int_1^2 1.5e^{-jk\pi t}dt \\ &= \frac{3}{2k\pi j}[1 - e{-}jk\pi] \\ &= \frac{3}{k\pi}e^{-jk(\pi/2)}\sin(\frac{k\pi}{2}) \end{aligned}$$

3.5. Both $x_1(1-t)$ and $x_1(t-1)$ are periodic with fundemental period $T_1 = \frac{2\pi}{\omega_1}$. Since $y(t)$ is a linear combination of $x_1(1-t)$ and $x_1(t-1)$, it is also periodic with fundemental period $T_2 = \frac{2\pi}{\omega_1}$. Therefore, $\omega_2 = \omega_1$.

Since $x_1(t) \stackrel{FS}{\longleftrightarrow} a_k$, using the results in Table 3.1 we have

$$x_1(t+1) \stackrel{FS}{\longleftrightarrow} a_k e^{jk(2\pi/T_1)}$$

$$x_1(t-1) \stackrel{FS}{\longleftrightarrow} a_k e^{-jk(2\pi/T_1)} \Rightarrow x_1(-t+1) \stackrel{FS}{\longleftrightarrow} a_{-k}e^{-jk(2\pi/T_1)}$$

Therefore,

$$x_1(t+1) + x_1(1-t) \stackrel{FS}{\longleftrightarrow} a_k e^{jk(2\pi/T_1)} + a_{-k}e^{-jk(2\pi/T_1)} = e^{-j\omega_1 k}(a_k + a_{-k})$$

3.6. (a) Comparing $x_1(t)$ with the Fourier series synthesis eq. (3.38), we obtain the Fourier series coefficients of $x_1(t)$ to be

$$a_k = \begin{cases} \left(\frac{1}{2}\right)^k, & 0 \leq k \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

From Table 3.1 we know that if $x_1(t)$ is real, then a_k has to be conjugate-symmetric, i.e, $a_k = a^*_{-k}$. Since this is not true for $x_1(t)$, the signal is **not real valued.**

Similarly, the Fourier series coefficients of $x_2(t)$ are

$$a_k = \begin{cases} \cos(k\pi), & 100 \leq k \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

From Table 3.1 we know that if $x_2(t)$ is real, then a_k has to be conjugate-symmetric, i.e, $a_k = a^*_{-k}$. Since this is true for $x_2(t)$, the signal is **real valued.**

Similarly, the Fourier series coefficients of $x_3(t)$ are

$$a_k = \begin{cases} j\sin(k\pi/2), & 100 \leq k \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

From Table 3.1 we know that if $x_3(t)$ is real, then a_k has to be conjugate-symmetric, i.e, $a_k = a^*_{-k}$. Since this is true for $x_3(t)$, the signal is **real valued.**

(b) For a signal to be even, its Fourier series coefficients must be even. This is true only for $x_2(t)$.

3.22. **(b)** $T = 2$, $a_k = \frac{-1^k}{2(1+jk\pi)}[e - e^{-1}]$ for all k.

3.26. **(a)** If $x(t)$ is real, then $x(t) = x^*(t)$. This implies that for $x(t)$ real $a_k = a^*_{-k}$. Since this is not true in this case problem, $x(t)$ is not real.

(b) If $x(t)$ is even, then $x(t) = x(-t)$ and $a_k = a_{-k}$. Since this is true for this case, $x(t)$ is even.

(c) We have

$$g(t) = \frac{dx(t)}{dt} \overset{FS}{\longleftrightarrow} b_k = jk\frac{2\pi}{T_0}a_k.$$

Therefore,

$$b_k = \begin{cases} 0, & k = 0 \\ -k(1/2)^{|k|}(2\pi/T_0), & \text{otherwise} \end{cases}.$$

Since b_k is not even, $g(t)$ is not even.

3.30. **(a)** The nonzero FS coefficients of $x(t)$ are $a_0 = 1$, $a_1 = a_{-1} = 1/2$.

(b) The nonzero FS coefficients of $x(t)$ are $b_1 = b^*_{-1} = e^{-j\pi/4}/2$.

(c) Using the multiplication property, we know that

$$z[n] = x[n]y[n] \overset{FS}{\longleftrightarrow} c_k = \sum_{l=-2}^{2} a_l b_{k-l}.$$

This implies that the nonzero Fourier series coefficients of $z[n]$ are $c_0 = \cos(\pi/4)/2$, $c_1 = c^*_{-1} = e^{-j\pi/4}/2$, $c_2 = c^*_{-2} = e^{-j\pi/4}/4$.

(d) We have

$$\begin{aligned} z[n] &= \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)\cos\left(\frac{2\pi}{6}n\right) \\ &= \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right) + \frac{1}{2}\left[\sin(\frac{4\pi}{6}n + \frac{\pi}{4}) + \sin(\frac{\pi}{4})\right] \end{aligned}$$

This implies that the nonzero Fourier series coefficients of $z[n]$ are $c_0 = \cos(\pi/4)/2$, $c_1 = c^*_{-1} = e^{-j\pi/4}/2$, $c_2 = c^*_{-2} = e^{-j\pi/4}/4$.

3.31. **(a)** $g[n]$ is as shown in Figure S3.31. Clearly, $g[n]$ has a fundamental period of 10.

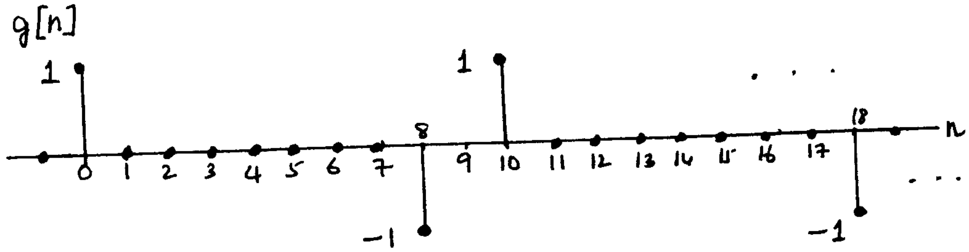

Figure S3.31

(b) The Fourier series coefficiennts of $g[n]$ are $b_k = (1/10)[1 - e^{-j(2\pi/10)8k}]$.

(c) Since $g[n] = x[n] - x[n-1]$, the FS coeffcients a_k and b_k must be related as

$$b_k = a_k - e^{-j(2\pi/10)k} a_k.$$

Therefore,

$$a_k = \frac{b_k}{1 - e^{-j(2\pi/10)k}} = \frac{(1/10)[1 - e^{-j(2\pi/10)8k}]}{1 - e^{-j(2\pi/10)k}}.$$