1-10 FIND THE FUNDAMENTAL PERIOD OF $x(t) = \underbrace{2\cos(10t+1)}_{} - \underbrace{\sin(4t-1)}_{}$

$10T_1 = 2\pi$ $\quad\quad$ $4T_2 = 2\pi$

$T_1 = \frac{\pi}{5}$ $\quad\quad$ $T_2 = \frac{\pi}{2}$

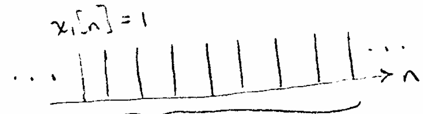
The least common multiple of T_1 and T_2 is π

$\therefore$ fundamental period $T = \pi$

1-11 FIND THE FUNDAMENTAL PERIOD OF $x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2\pi n}{5}}$

$\boxed{N_1 = 1}$

$x_1[n] = 1$



repeats every sample, so $N_1 = 1$

$\frac{4\pi N_2}{7} = m(2\pi)$

for $m = 1$, $N_2 = \frac{7}{2} \rightarrow$ not a valid value

for $m = 2$, $\boxed{N_2 = 7}$

$\frac{2\pi N_3}{5} = m(2$

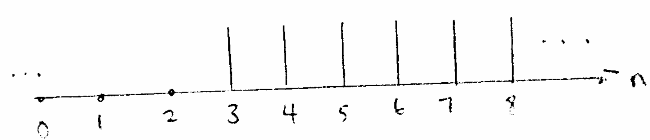
for $m = 1$,

$\boxed{N_3 = 5}$

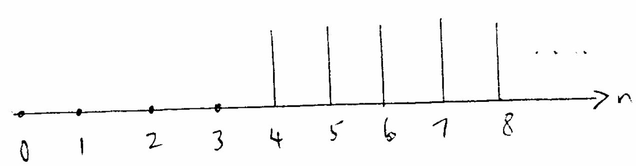
$\therefore$ The fundamental period of $x[n]$ is 35.

1-12 Express $x[n] = 1 - \sum_{k=3}^{\infty} \delta[n-1-k]$ in the form $x[n] = u[Mn - n_0]$

$\sum_{k=3}^{\infty} \delta[n-k]$:

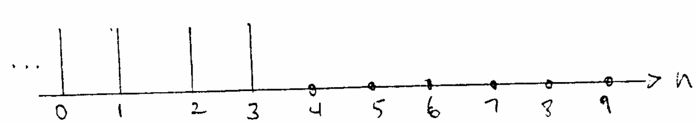


$\sum_{k=3}^{\infty} \delta[n-k-1]$:



(shifted right by 1

$1 - \sum_{k=3}^{\infty} \delta[n-k-1]$

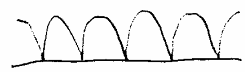


$u[3-n] \longrightarrow$ $\boxed{\begin{matrix} M = -1 \\ n_0 = -3 \end{matrix}}$

1-25 DETERMINE IF THE FOLLOWING SIGNALS ARE PERIODIC, AND IF SO FIND THEIR FUNDAMENTAL PERIODS.

A) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right) \longrightarrow$ PERIODIC $\quad 4T = 2\pi$, $T = \frac{\pi}{2}$

B) $x(t) = e^{j(\pi t - 1)} \longrightarrow$ PERIODIC $\quad \pi T = 2\pi$, $T = 2$

c) $x(t) = \cos^2\left(2t - \frac{\pi}{3}\right) \longrightarrow$ HAS SHAPE  SO PERIODIC.

PERIOD : $2T = \frac{2\pi}{2}$ (PERIOD WILL BE HALF OF CALCULATED VALUE BECAUSE SQUARE WAS TAKEN)

$T = \frac{\pi}{2}$

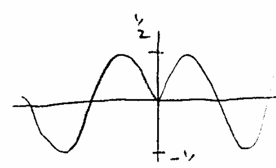
ALTERNATE SOLUTION: USE $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$x(t) = \cos^2\left(2t - \frac{\pi}{3}\right) = \frac{1}{2}\left(1 + \cos\left(2\left(2t - \frac{\pi}{3}\right)\right)\right) = \underbrace{\frac{1}{2}\left(1 + \cos\left(4t - \frac{2\pi}{3}\right)\right)}_{4T = 2\pi,\ T = \frac{\pi}{2}}$$

D) $x(t) = \mathcal{E}v\{\cos(4\pi t)u(t)\} = \dfrac{\cos(4\pi t)u(t) + \cos(-4\pi t)u(-t)}{2}$

$= \frac{1}{2}\cos(4\pi t) \quad \therefore$ PERIODIC WITH $T = \frac{1}{2}$

E) $x(t) = \mathcal{E}v\{\sin(4\pi t)u(t)\} = \dfrac{\sin(4\pi t)u(t) + \sin(-4\pi t)u(-t)}{2}$



There is a discontinuity at $t=0$ $\therefore$ not periodic

F) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)} u(2t-n)$

This is an infinite sum of half-infinite, exponentially decaying signals

$\rightarrow$ NOT PERIODIC

(1-26) Determine if the following signals are periodic, and if so find their fundamental periods.

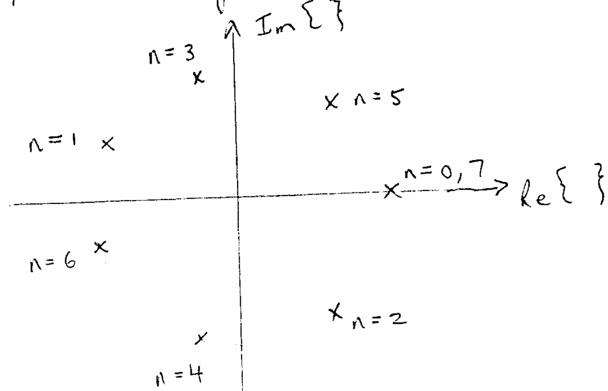
A) $x[n] = \sin\left[\frac{6\pi n}{7} + 1\right]$

PERIODIC (because the frequency is a rational multiple of π)

$$\frac{6\pi N}{7} = m(2\pi) \qquad N, m \in \mathbb{N}$$

$$N = \frac{7}{3}m \rightarrow \boxed{N = 7} \text{ if } m = 3$$

$m = 3$ means that the signal rotates around the complex plane 3 times before returning to its original value:



B) $x[n] = \cos\left[\frac{n}{8} - \pi\right]$

↳ try to find a period: $\frac{N}{8} = m(2\pi)$

$$\Rightarrow N = m(16\pi)$$

Clearly, no integer m will give an integer value of N, ∴ NOT PERIODIC

C) $x[n] = \cos\left[\frac{\pi}{8}n^2\right]$ ⇝ if you work this out it actually turns out to be periodic with $N = 8$, but this is a more difficult question than those you are expected to be able to solve.

D) $x[n] = \underbrace{\cos\left[\frac{\pi n}{2}\right]} \underbrace{\cos\left[\frac{\pi n}{4}\right]}$

$$\frac{\pi N_1}{2} = m(2\pi) \qquad \frac{\pi N_2}{4} = m(2\pi)$$

$$N_1 = 4m \qquad N_2 = 8m$$

$$N_1 = 4 \quad (m=1) \qquad N_2 = 8 \quad (m=1)$$

The least common multiple of N_1 and N_2 is 8, $\therefore$ $x[n]$ is <u>periodic</u> with $N = 8$.

E) $x[n] = \underbrace{2\cos\left[\frac{\pi n}{4}\right]} + \underbrace{\sin\left[\frac{\pi n}{8}\right]} - \underbrace{2\cos\left[\frac{\pi n}{2} + \frac{\pi}{6}\right]}$

$$\frac{\pi N_1}{4} = m(2\pi) \qquad \frac{\pi N_2}{8} = m(2\pi) \qquad \frac{\pi N_3}{2} = m(2\pi)$$

$$N_1 = 8m \qquad N_2 = 16m \qquad N_3 = 4m$$

$$N_1 = 8 \text{ if } m=1 \qquad N_2 = 16 \quad (m=1) \qquad N_3 = 4 \quad (m=1)$$

$\Rightarrow$ $\therefore$ $x[n]$ is <u>periodic</u> with $N = 16$.