1.19

- (a) Suppose for input $x_1(t)$ and $x_2(t)$, we have the output

$$y_1(t) = t^2 x_1(t-1)$$

and

$$y_2(t) = t^2 x_2(t-1).$$

Let

$$x_3(t) = ax_1(t) + bx_2(t),$$

then

$$\begin{aligned} y_3(t) &= t^2 x_3(t-1) \\ &= t^2(ax_1(t-1) + bx_2(t-1)) \\ &= ay_1(t) + by_2(t). \end{aligned}$$

So the system is linear.
Next if we have

$$x_4(t) = x_1(t-t_0),$$

then

$$\begin{aligned} y_4(t) &= t^2 x_4(t-1) \\ &= t^2 x_1(t-t_0-1). \end{aligned}$$

However, for system to be time-invariant,

$$y_1(t-t_0) = (t-t_0)^2 x_1(t-t_0-1).$$

So the system is not time-invariant.

- (b) Suppose for input $x_1[n]$ and $x_2[n]$, we have the output

$$y_1[n] = x_1^2[n-2]$$

and

$$y_2[n] = x_2^2[n-2].$$

Let

$$x_3[n] = ax_1[n] + bx_2[n],$$

then

$$\begin{aligned} y_3[n] &= x_3^2[n-2] \\ &= (ax_1[n-2] + bx_2[n-2])^2 \\ &\neq ay_1[n] + by_2[n]. \end{aligned}$$

So the system is not linear.
Next if we have

$$x_4[n] = x_1[n-n_0],$$

then

$$\begin{aligned} y_4[n] &= x_4^2[n-2] \\ &= x_1^2[n-n_0-2]. \end{aligned}$$

For system to be time-invariant,

$$y_1[n-n_0] = x_1^2[n-n_0-2].$$

So the system is time-invariant.

- (c) Suppose for input $x_1[n]$ and $x_2[n]$, we have the output

$$y_1[n] = x_1[n+1] - x_1[n-1]$$

and

$$y_2[n] = x_2[n+1] - x_2[n-1].$$

Let

$$x_3[n] = ax_1[n] + bx_2[n],$$

then

$$\begin{aligned} y_3[n] &= x_3[n+1] - x_3[n-1] \\ &= ax_1[n+1] + bx_2[n+1] - ax_1[n-1] - bx_2[n-1] \\ &= ay_1[n] + by_2[n]. \end{aligned}$$

So the system is linear.

Next if we have

$$x_4[n] = x_1[n-n_0],$$

then

$$\begin{aligned} y_4[n] &= x_4[n+1] - x_4[n-1] \\ &= x_1[n-n_0+1] - x_1[n-n_0-1]. \end{aligned}$$

For system to be time-invariant,

$$y_1[n-n_0] = x_1[n-n_0+1] - x_1[n-n_0-1].$$

So the system is time-invariant.

- (d) Suppose for input $x_1(t)$ and $x_2(t)$, we have the output

$$y_1(t) = \mathcal{O}d\{x_1(t)\}$$

and

$$y_2(t) = \mathcal{O}d\{x_2(t)\}.$$

Let

$$x_3(t) = ax_1(t) + bx_2(t),$$

then

$$\begin{aligned} y_3(t) &= \mathcal{O}d\{x_3(t)\} \\ &= \mathcal{O}d\{ax_1(t) + bx_2(t)\} \\ &= a\mathcal{O}d\{x_1(t)\} + b\mathcal{O}d\{x_2(t)\} \\ &= ay_1(t) + by_2(t). \end{aligned}$$

So the system is linear.

Next if we have

$$x_4(t) = x_1(t-t_0),$$

then

$$\begin{aligned} y_4(t) &= \mathcal{O}d\{x_4(t)\} \\ &= (x_4(t) - x_4(-t))/2 \\ &= (x_1(t-t_0) - x_1(-t-t_0))/2. \end{aligned}$$

However, for system to be time-invariant,

$$y_1(t-t_0) = (x_1(t-t_0) - x_1(-t+t_0))/2.$$

So the system is not time-invariant.

1.28

- (a) Linear and stable
- (b) Linear, time-invariant, causal, and stable
- (c) Linear, memoryless, and causal
- (d) Linear and stable
- (e) Linear and stable
- (f) Linear, memoryless, causal, and stable
- (g) Linear and stable

1.26

- (c) Suppose $x[n] = x[n+N]$, then

  $$\cos[\frac{\pi}{8}n^2] = \cos[\frac{\pi}{8}(n+N)^2].$$

  Thus,

  $$\frac{\pi}{8}n^2 + 2\pi k = \frac{\pi}{8}(n^2 + N^2 + 2nN)$$

  $$\Rightarrow k = \frac{nN}{8} + \frac{N^2}{16}.$$

  For $x[n]$ to be periodic, k must be an integer for all n. So the smallest $N = 8$.