Solution for ELG 3120 Assignment #4

2.19. **(a)** Consider the difference equation relating $y[n]$ and $w[n]$ for S_2:

$$y[n] = \alpha y[n-1] + \beta w[n]$$

From this we may write

$$w[n] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1]$$

and

$$w[n-1] = \frac{1}{\beta}y[n-1] - \frac{\alpha}{\beta}y[n-2]$$

Weighting the previous equation by 1/2 and subtracting from the one before, we obtain

$$w[n] - \frac{1}{2}w[n-1] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2]$$

Substituting this in the difference equation relating $w[n]$ and $x[n]$ for S_1,

$$\frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2] = x[n]$$

That is,

$$y[n] = (\alpha + \frac{1}{2})y[n-1] - \frac{\alpha}{2}y[n-2] + \beta x[n]$$

Comparing with the given equation relating $y[n]$ and $x[n]$, we obtain

$$\alpha = \frac{1}{4}, \qquad \beta = 1$$

(b) The difference equations relating the input and output of the systems S_1 and S_2 are

$$w[n] = \frac{1}{2}w[n-1] + x[n] \quad \text{and} \quad y[n] = \frac{1}{4}y[n-1] + w[n]$$

From these, we can use the method specifed in Example 2.15 to show that the impulse responses of S_1 and S_2 are

$$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_2[n] = \left(\frac{1}{4}\right)^n u[n],$$

respectively. The overall impulse response of the system made up of a cascade of S_1 and S_2 will be

$$\begin{aligned} h[n] &= h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k]h_2[n-k] \\ &= \sum_{k=0}^{\infty}(\frac{1}{2})^k(\frac{1}{4})^{n-k}u[n-k] \\ &= \sum_{k=0}^{n}(\frac{1}{2})^k(\frac{1}{4})^{n-k} = \sum_{k=0}^{n}(\frac{1}{2})^{2(n-k)} \\ &= [2(\frac{1}{2})^n - (\frac{1}{4})^n]u[n] \end{aligned}$$

2.28

(b) Noncausal because $h[n] \neq 0$ for $n < 0$

Stable because $\sum_{n=-2}^{\infty} (0.8)^n = \frac{125}{16} < \infty$

(e) Causal because $h[n] = 0$ for $n < 0$

Unstable because the second term becomes infinite as $n \to \infty$

2.29

(d) Noncausal because $h(t) \neq 0$ for $t < 0$

Stable because $\int_{-\infty}^{\infty} |h(t)| dt = \frac{e^{-2}}{2} < \infty$

(e) Causal because $h(t) = 0$ for $t < 0$

Stable because $\int_{-\infty}^{\infty} |h(t)| dt = \frac{1}{3} < \infty$

(f) Causal since h(t)=0, for t<0

Stable since

$$\int_{-\infty}^{+\infty} |te^{-t}u(t)| dt = \int_{0}^{+\infty} |te^{-t}| dt = \left(-te^{-t} - e^{-t}\right)\Big|_0^{\infty} = 1 < \infty$$

2.31

Initial rest implies that $y[n] = 0$ for $n < -2$. Now

$y[n] = x[n] + 2x[n-2] - 2y[n-1]$

so $y[-2] = 1$

$y[-1] = 0$

$y[0] = 5$

$y[1] = -4$

$y[2] = 16$

$y[3] = -27$

$y[4] = 58$

$y[5] = -114$

$y[n] = (-2)^{n-5}\, y[5] = -114(-2)^{n-5}$

2.32

(a) If $y_h[n] = A\left(\frac{1}{2}\right)^n$, then we need to verify $A\left(\frac{1}{2}\right)^n - \frac{1}{2}A\left(\frac{1}{2}\right)^{n-1} = 0$

Clearly this is true.

(b) We now require that for $n \geq 0$

$B\left(\frac{1}{3}\right)^n - \frac{1}{2}B\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$, so $B = -2$

(c) From eq. (P2.32-1), we know that $y[0] = x[0] + \frac{1}{2} y[-1] = x[0] = 1$, Now we also have $y[0] = A + B \Rightarrow A = 1 - B = 3$