Initials _________

Université d'Ottawa · University of Ottawa

Faculté de génie
École d'ingénierie
et de technologie de l'information

Faculty of Engineering
School of Information Technology
and Engineering

ELG 3120C

Signals and Systems

Midterm Exam

Friday, 15 February 2001

Time: 10:00 am – 11:20 am

CBY B202

Prof. Jianping Yao

Calculator not allowed.
Textbook and notes not allowed (close book exam).

Last name:

First name:

Student number:

Question 1

Determine if the following systems are: causal, stable, time invariant and linear. Justify your answers.

(a) $y(t) = x(t-1) + x(2-t)$ (10 marks)

(b) $y[n] = [\cos(3n)]x[n]$ (10 marks)

Question 2

(1) Calculate the following convolution: $y(t) = x(t) * h(t)$ with $x(t) = e^{-at}u(t)$, $a > 0$ and $h(t) = u(t) - u(t-4)$. (20 marks)

(2) Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related by the difference equation

$$y[n] = \frac{1}{4} y[n-1] + x[n]$$

Determine $y[n]$ if $x[n] = u[n] - u[n-2]$. (20 marks)

Question 3

(1) Let $x(t) = \begin{cases} t & 0 \le t \le 1 \\ -t & -1 \le t \le 0 \end{cases}$ be a periodic signal with fundamental period of $T = 2$ and Fourier series coefficients a_k.

(a) Sketch the waveform of $x(t)$ and $dx(t)/dt$.

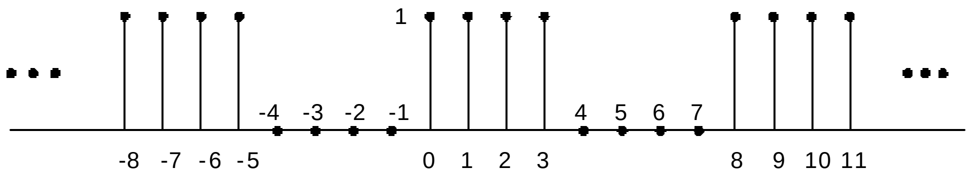
(b) Calculate a_0.

(c) Determine the Fourier series representation of $g(t) = dx(t)/dt$.

(d) Using the results from Part (c) and the property of continuous-time Fourier series to determine the Fourier series coefficients of $x(t)$.

(20 marks)

(2) Determine the Fourier series coefficients for the following discrete-time periodic signal.



(20 marks)

Convolutions:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$x(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t)h(t-t)dt$$

The Fourier series of a periodic continuous-time signal:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2p/T)t}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jkw_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2p/T)t}dt$$

The Fourier series of a periodic discrete-time signal:

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2p/N)n}$$

$$a_k = \frac{1}{N}\sum_{k=\langle N \rangle} x[n]e^{-jk(2p/N)n}$$

Properties of the Continuous-Time Fourier Series

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamenta l frequency $w_0 = 2p / T$	a_k b_k
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	$x(t - t_0)$	$e^{-jkw_0 t} a_k$
Frequency shifting	$e^{jMw_0 t} x(t)$	a_{k-M}
Conjugation	$x^*(t)$	a^*_{-k}
Time Reversal	$x(-t)$	a_{-k}
Time Scaling	$x(at)$, $a > 0$ (Periodic with period T / a)	a_k
Periodic Convolution	$\int_T x(t) y(t - t) dt$	$Ta_k b_k$
Multiplication	$x(t) y(t)$	$\sum_{l=-\infty}^{\infty} a_l b_{k-l}$
Differentiation	$\frac{dx(t)}{dt}$	$jkw_0 a_k = jk \frac{2p}{T} a_k$
Integration	$\int_{-\infty}^{t} x(t) dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jkw_0}\right) a_k = \left(\frac{1}{jk(2p / T)}\right) a_k$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathrm{Re}\{a_k\} = \mathrm{Re}\{a_{-k}\} \\ \mathrm{Im}\{a_k\} = -\mathrm{Im}\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals Real and Odd Signals Even-Odd Decomposition of Real Signals	$x(t)$ real and even $x(t)$ real and odd $\begin{cases} x_e(t) = Ev\{x(t)\} & [x(t)\ real] \\ x_e(t) = Od\{x(t)\} & [x(t)\ real] \end{cases}$	a_k real and even a_k purely imaginary and odd $\mathrm{Re}\{a_k\}$ $j\,\mathrm{Im}\{a_k\}$
	Parseval's Relation for Periodic Signals $\frac{1}{T}\int_T \lvert x(t) \rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert a_k \rvert^2$	