ELG3120A Midterm Solutions

Question 1

(a) $y(t) = (t+5)\cos(\frac{1}{x(t)})$

1.

Since the output depends on the present value of the input, the system is causal.

2.

$t \to \infty$

$y(t) = (t+5)\cos(\frac{1}{x(t)}) \to \infty$

Bounded input generate unbounded output, the system is not stable.

3.

The I/O equation of the system

$$y(t) = (t+5)\cos(\frac{1}{x(t)}) = t\{x(t)\}$$

The response of this system to $x(t-t_0)$ is

$$y(t,t_0) = (t+5)\cos(\frac{1}{x(t-t_0)}) \qquad (1)$$

We delay the output by t_0 time, we obtain

$$y(t-t_0) = (t-t_0+5)\cos(\frac{1}{x(t-t_0)}) \qquad (2)$$

Since

$$y(t,t_0) \neq y(t-t_0)$$

The system is time variant.

4.

For two input sequences $x_1(t)$ and $x_2(t)$, the corresponding outputs are

$$y_1(t) = (t+5)\cos(\frac{1}{x_1(t)})$$

$$y_2(t) = (t+5)\cos(\frac{1}{x_2(t)})$$

A linear combination of the two inputs sequences results in the output

$$y_3(t) = t\,[ax_1(t) + bx_2(t)] = (t+5)\cos(\frac{1}{ax_1(t)}) + (t+5)\cos(\frac{1}{bx_2(t)}) \qquad (3)$$

A linear combination of the output results in the output:

$$ay_1(t) + by_2(t) \neq (t+5)\cos(\frac{1}{ax_1(t)}) + (t+5)\cos(\frac{1}{bx_2(t)}) \qquad (4)$$

Since the right sides of (3) and (4) are not identical, the system is not linear.

b) y[n]=log(x[2n-4])

1.

Since the output depends on the future values of the input when (n>4), the system is not causal.

2.

$$when\left|x[n]\right| \to 0$$
$$\left|y[n]\right| = \left|\log(x[2n-4])\right| \to \infty$$

Bounded input generates unbounded output, the system is not stable.

3.

$$y[n] = \log(x[2n-4]) = t\,\{x[n]\}$$

If the input is delayed by n_0 units in time and applied to the system, the output will be

$$\mathrm{y}[\mathrm{n}, \mathrm{n}_0] = \log(x[2n - n_0 - 4]) \qquad (5)$$

We delay y[n] by n_0 units in time, we obtain

$$\mathrm{y}[\mathrm{n} - \mathrm{n}_0] = \log(x[2(n - n_0) - 4]) \qquad (6)$$

Since the right sides of (5) and (6) are identical,

$$\mathrm{y}[\mathrm{n}, \mathrm{n}_0] \neq \mathrm{y}[\mathrm{n} - \mathrm{n}_0]$$

The system is time variant.

4.

For two input sequences $x_1[n]$ and $x_2[n]$, the corresponding outputs are

$$y_1[n] = \log(x_1[2n-4])$$

$$y_2[n] = \log(x_2[2n-4])$$

A linear combination of the two inputs sequences results in the output

$$y_3[n] = t\{ax_1[n] + bx_2[n]\} = \log(ax_1[2n-4]) + \log(bx_2[2n-4]) \tag{7}$$

A linear combination of the outputs results in the output

$$ay_1[n] + by_2[n] = a\{\log(x_1[2n-4]\} + b\{\log(x_2[2n-4]\} \tag{8}$$

Since the right sides of (7) and (8) are not identical, the system is not linear.

Question 2

(a)With $d(t)$ as input, we can get y(t) as h(t).

$y(t) = y_h(t) + y_p(t)$

Because $x(t) = d(t)$

$\Rightarrow y_p(t) = 0$

$y_h(t) = Ae^{st}$

$4Ase^{st} - 2Ae^{st} = 0$

s=1/2

Then,

$$4\frac{dh(t)}{dt} - 2h(t) = d(t)$$

$$4\int_{0-}^{0+}\frac{dh(t)}{dt}dt - \int_{0-}^{0+}2h(t)dt = \int_{0-}^{0+}d(t)dt$$

$$\because \int_{0-}^{0+}2h(t)dt = 0$$

$$\therefore 4[h(t)]_{0-}^{0+} = 1$$

$$\therefore 4[h(0^+) - h(0^-)] = 1$$

$$\because h(0^-) = 0$$

$$\therefore 4h(0^+) = 1$$

$$4Ae^{\frac{1}{2}\cdot 0} = 1$$

$$\therefore A = \frac{1}{4}$$

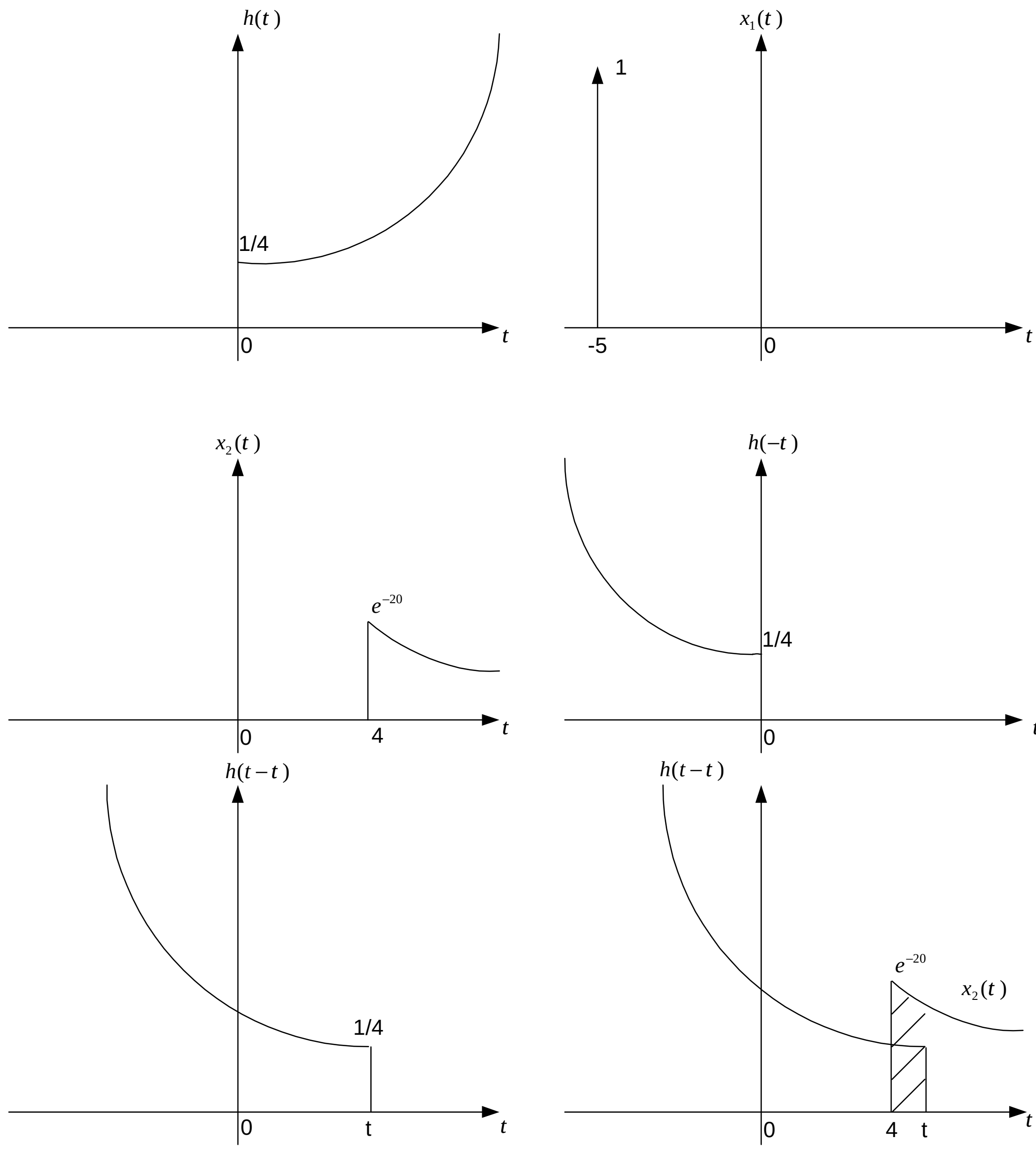
$$\therefore y(t) = h(t) = \frac{1}{4}e^{\frac{1}{2}t}u(t)$$

(b) For $y(t) = h(t) = \frac{1}{4}e^{\frac{1}{2}t}u(t)$

$$\int_{-\infty}^{\infty}h(t)dt = \int_{-\infty}^{\infty}\frac{1}{4}e^{\frac{1}{2}t}u(t)dt \to \infty$$

So, the system is not stable.

(c)



Because this is a LTI system, we can have

$$y(t) = x_1(t) * h(t) + x_2(t) * h(t)$$

Where $x_1(t) = d(t+5)$
$x_2(t) = e^{-5t} u(t-4)$

Thus, $y_1(t) = d(t+5) * h(t) = \frac{1}{4} e^{\frac{1}{2}(t+5)} u(t+5)$

$$for......t < 4, y_2(t) = 0$$

$$for......t \geq 4, y_2(t) = \frac{1}{4}\int_4^t e^{-5t} e^{\frac{1}{2}(t-t)} dt = -\frac{1}{22}(e^{-5} - e^{-22})u(t-4)$$

$$y(t) = y_1(t) + y_2(t) = \frac{1}{4} e^{\frac{1}{2}(t+5)} u(t+5) - \frac{1}{22}(e^{-5} - e^{-22})u(t-4)$$

Question 3

(a)

$$N = 6$$

$$a_k = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jk(\frac{2p}{N})n} = \frac{1}{6}\sum_{n=0}^{5} x[n] e^{-jk(\frac{2p}{6})n}$$

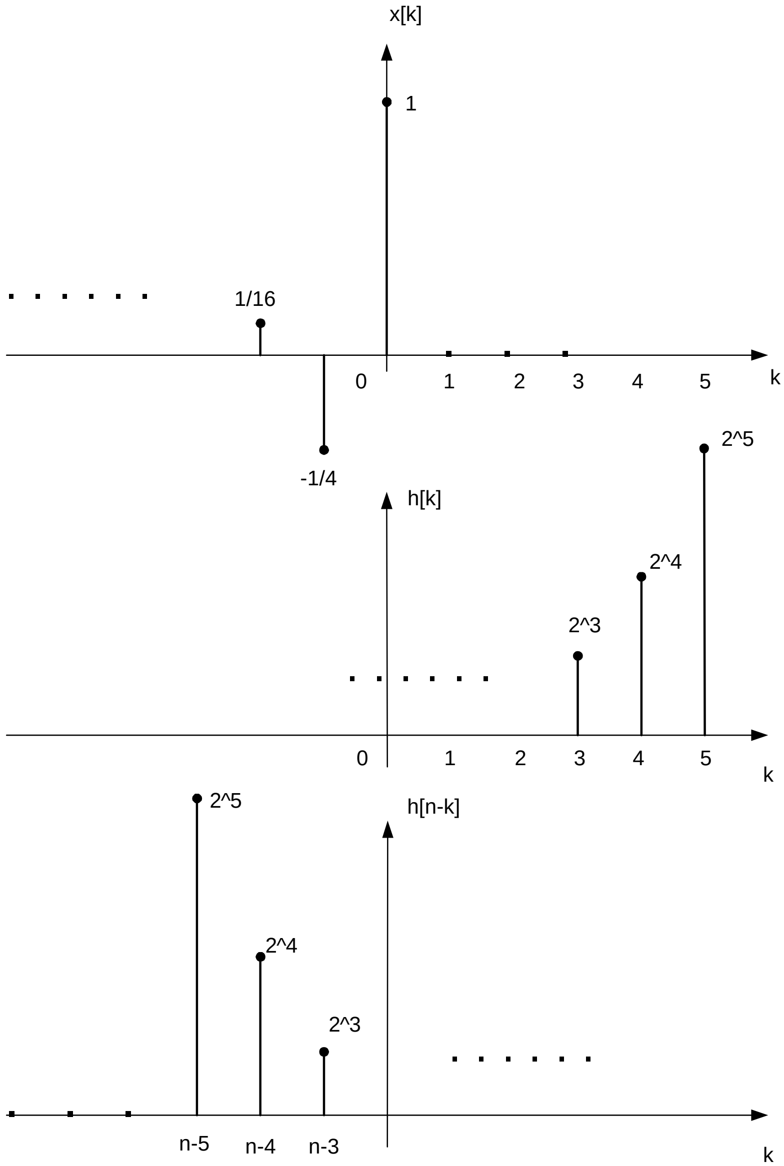
$$a_k = \frac{1}{6}(0 + 3e^{-jk(\frac{2p}{6})1} + 0 + 0 + 1e^{-jk(\frac{2p}{6})4} + 0) = \frac{1}{2}e^{-jk\frac{p}{3}} + \frac{1}{6}e^{-jk\frac{4p}{3}}$$

(b)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} where...w_0 = \frac{2p}{T} = p$$

So $x(t) = 10 + 2je^{j3pt} - 2je^{-j3pt} + 5e^{j5pt} + 5e^{-j5pt} = 10 - 4\sin 3pt + 10\cos 5pt$

Question 4



$for...n > 5$

$y[n] = 0$

$For...$ n =<5

$$y[n] = \sum_{k=n-5}^{0} x[k]h[n-k] = \sum_{k=n-5}^{0} (-4)^k 2^{n-k} = 2^n \sum_{k=n-5}^{0} (\frac{-4}{2})^k = 2^n \frac{(-2)^{n-5} - (-2)^{0+1}}{1-(-2)} = \frac{2^{n+1} - 2(-4)^{n-3}}{3}$$