

Université d'Ottawa
Faculté de Génie,
École d'Ingénierie et des
Technologies de l'Information



University of Ottawa
Faculty of Engineering,
School of Information
Technology and Engineering

ELG 3120 A

Signal and System Analysis

Midterm

Wednesday, 30 Octobre 2002

MCD 146, 10:00 am-11:25 am

Professor: Jianping Yao

- **No calculators**
- **No textbook and notes (close-book exam)**
- **Initial on the top of each page.**

Last name: _____

First name: _____

Student number: _____

Summations :

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \quad |a| < 1$$

$$\sum_{k=n_1}^{\infty} a^k = \frac{a^{n_1}}{1-a} \quad |a| < 1$$

$$\sum_{k=0}^{n_1} a^k = \frac{1-a^{n_1+1}}{1-a} \quad a \neq 1$$

$$\sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1} - a^{n_2+1}}{1-a} \quad a \neq 1$$

Convolutions :

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\mathbf{t})h(t - \mathbf{t})d\mathbf{t}$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$

Continuous-time Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

$$a_0 = \frac{1}{T} \int_T x(t) dt \quad \omega_0 = \frac{2\pi}{T}$$

Discrete-time Fourier Series:

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

Question 1 :**/5**

Determine if the following systems are: causal or non-causal, stable or instable, time invariant or time variant, and linear or nonlinear. Justify your answers.

(a) $y(t) = (t + 5) \cos\left(\frac{1}{x(t)}\right)$

(b) $y[n] = \log(x[2n - 4])$

Question 2 :**/8**

A causal LTI system is described by the following differential equation:

$$4 \frac{dy(x)}{dt} - 2y(t) = x(t)$$

- (a) Calculate the impulse response $h(t)$ of the system by solving the differential equation. (Do not use the transforms for this question, such as Fourier transform or Laplace transform).
- (b) Based on $h(t)$, determine if the system is stable or not.
- (c) If the input is $x(t) = \mathbf{d}(t+5) + e^{-5t}u(t-4)$, calculate the output $y(t)$ using convolution $y(t) = x(t) * h(t)$. (Note: if you cannot find $h(t)$ in (a), then you may use $h(t) = e^t u(t)$ to calculate the convolution).

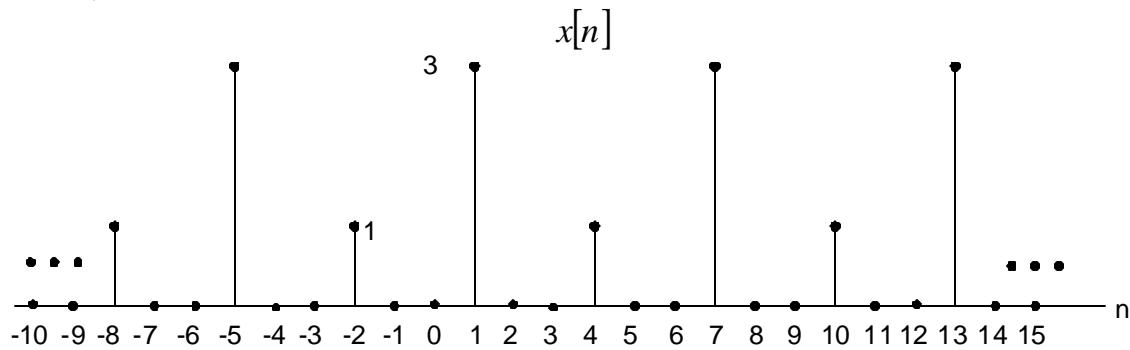
Initials _____

Question 3 :

/6

(a) Calculate the Fourier Series coefficients of the following discrete-time signal.

$$x[n] = \sum_{m=-\infty}^{+\infty} \mathbf{d}(n - 6m - 4) + 3\mathbf{d}(n - 6m - 1)$$



(b) Calculate the continuous-time signal $x(t)$ with fundamental period $T = 2$. Find the expression which is pure real (i.e. simplified). The Fourier series coefficients a_k are:

$$a_0 = 10$$

$$a_3 = 2j$$

$$a_{-3} = -2j$$

$$a_5 = 5$$

$$a_{-5} = 5$$

$$a_k = 0 \quad \text{others}$$

Initials _____

Question 4 :**/6**

Calculate the convolution sum $y[n] = x[n] * h[n]$ with $x[n] = (-4)^n u[-n]$ and $h[n] = 2^n u[-n + 5]$.

Initials _____