Question 1

1.1

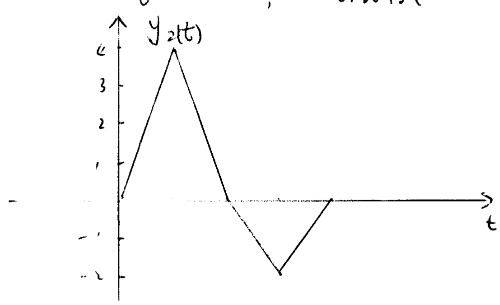
$$x_2(t) = 2x_1(t) - x_1(t-2)$$

$\because$ This is an LTI system

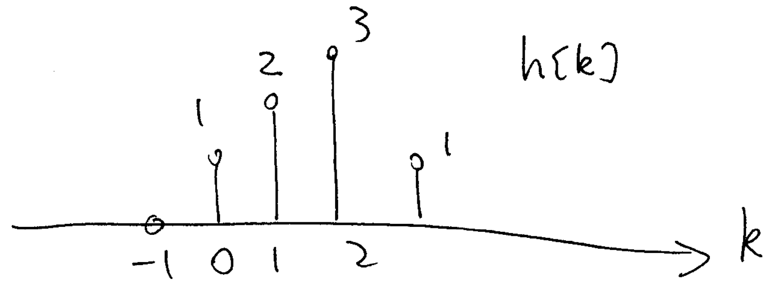
$$\therefore \quad y_2(t) = x_2(t) * h(t)$$
$$= \left[2x_1(t) - x_1(t-2)\right] * h(t)$$
$$= 2y_1(t) - y_1(t-2)$$

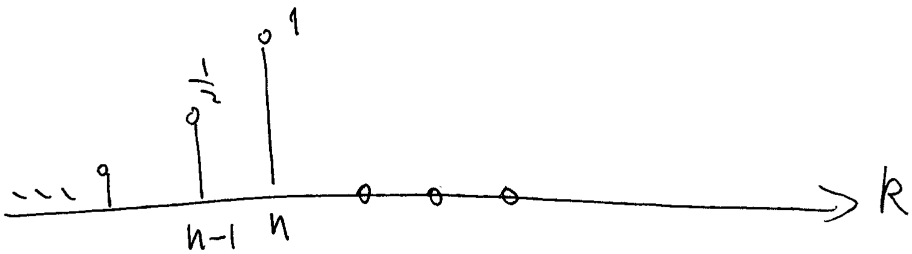
$\therefore$

$$y_2(t) = \begin{cases} 4t\,; & 0 \le t \le 1 \\ 8-4t\,; & 1 < t \le 2 \\ -2(t-2)\,; & 2 < t \le 3 \\ -[8-4(t-2)]\,; & 3 < t \le 4 \\ 0\,; & \text{otherwise} \end{cases}$$



1.2





$$y[n] = \begin{cases} 0 & n < 0 \\ \sum_{k=0}^{n} h[k] \left(\frac{1}{2}\right)^{n-k} & 0 \le n < 3 \\ \sum_{k=0}^{3} h[k] \left(\frac{1}{2}\right)^{n-k} & n \ge 3 \end{cases}$$

$$= \begin{cases} 0 & n < 0 \\ 2\frac{1}{2} & n = 1 \\ 4\frac{1}{4} & n = 2 \\ 25 \cdot \left(\frac{1}{2}\right)^{n} & n \ge 3 \end{cases}$$

$\mathfrak{X}$ 2.1

(3)

$$X_0(j\omega) = \int_0^1 e^{-t} e^{-j\omega t} dt = \int_0^1 e^{-(1+j\omega)t} dt$$

$$= -\frac{e^{-(j\omega+1)}}{j\omega+1} + \frac{1}{j\omega+1}$$

$$x_0(t) \xleftrightarrow{FT} X_0(j\omega)$$

$$\therefore \quad t\,x_0(t) \xleftrightarrow{FT} j\frac{d}{d\omega} X_0(j\omega)$$

$$j\frac{d}{d\omega} X_0(j\omega) = j\frac{d}{d\omega}\left(-\frac{e^{-(j\omega+1)}}{j\omega+1} + \frac{1}{j\omega+1}\right)$$

$$= \frac{1-2e^{-1}e^{-j\omega} - j\omega e^{-1}e^{-j\omega}}{(1+j\omega)^2}$$

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(4)

2.2. $G(j\omega) = \begin{cases} 1 ; & |\omega| \le 2 \\ 0 ; & \text{otherwise} \end{cases} \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{\sin 2t}{\pi t}$

$\therefore \; x(t) = \dfrac{\sin 2t}{\pi t \cdot \cos t} = \dfrac{2 \sin t}{\pi t}$.

Question 3.

a. $$\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

b. $$H(j\omega) = \frac{2}{2+j\omega} + \frac{-1}{3+j\omega}$$

$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$$

c. $$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$= \left(\frac{1}{4+j\omega} - \frac{1}{(4+j\omega)^2}\right)\left(\frac{4+j\omega}{(2+j\omega)(3+j\omega)}\right)$$

$$= \frac{-\frac{1}{2}}{4+j\omega} + \frac{\frac{1}{2}}{2+j\omega}$$

$$\therefore\ y(t) = -\frac{1}{2}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t)$$

Question 4.

a. $H(e^{jw}) = H_1(e^{jw}) \cdot H_2(e^{jw}) = \frac{1}{1+\frac{1}{2}e^{-jw}} + \frac{1}{1-\frac{1}{2}e^{-jw}}$

$\therefore h[n] = (-\frac{1}{2})^n u[n] + (\frac{1}{2})^n u[n]$

b. $\left| \sum_n (-\frac{1}{2})^n u[n] + \sum_n (\frac{1}{2})^n u[n] \right| < \infty$

$\therefore$ the system is stable.

c. $X(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}}$

$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$

$= -\frac{8}{5}\left(\frac{1}{1-\frac{1}{3}e^{-jw}}\right) + \frac{3}{5}\left(\frac{1}{1+\frac{1}{2}e^{-jw}}\right) + 3\left(\frac{1}{1-\frac{1}{2}e^{-jw}}\right)$

$\therefore y[n] = -\frac{8}{5}(\frac{1}{3})^n u[n] + \frac{3}{5}(-\frac{1}{2})^n u[n] + 3(\frac{1}{2})^n u[n]$

Question 5.

a. $LC\dfrac{d^2 y(t)}{dt^2} + RC\dfrac{dy(t)}{dt} + y(t) = x(t)$

That is $\dfrac{d^2 y(t)}{dt^2} + \dfrac{dy(t)}{dt} + y(t) = x(t)$

b. $H(s) = \dfrac{1}{1+s+s^2}$

$\therefore h(t) = \dfrac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cdot \sin\frac{\sqrt{3}}{2}t$

c. $\dfrac{d^2 y(t)}{dt^2} + 2\xi\omega_n\dfrac{dy(t)}{dt} + y(t) = x(t)$

When $R \geq 2\sqrt{\frac{L}{C}} = 2\Omega$

there is no oscillation

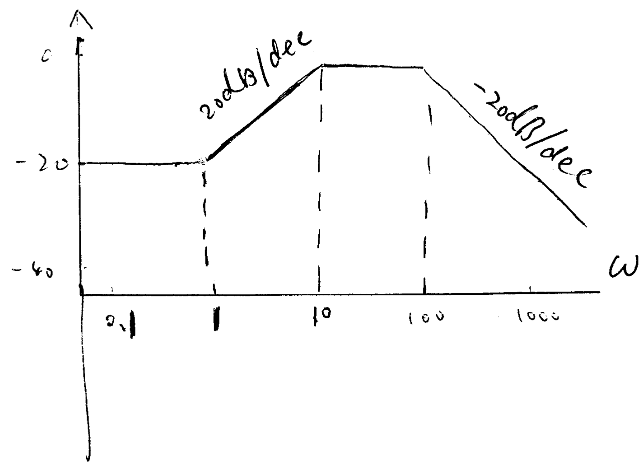
8

Question 6

6.1

$$|H(jw)| = \frac{1}{10}\left(\frac{1}{1+jw/10}\right)\left(\frac{1}{1+jw/100}\right)(1+jw)$$

$20\log_{10}|H(j\omega)|$



6.2 $Y(e^{jw}) - a e^{-jw} Y(e^{jw}) = X(e^{jw})$

$$H(e^{jw}) = \frac{1}{1 - a e^{-jw}}$$

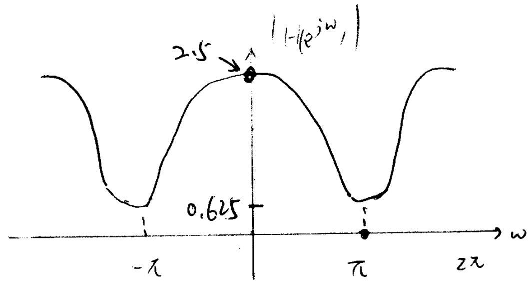
$$|H(e^{jw})| = \left| \frac{1}{1 - a e^{-jw}} \right| = \frac{1}{\sqrt{1 + a^2 - 2a\cos w}}$$

$a = 0.6$, $w = 0$

$$|H(e^{jw})| = \frac{1}{\sqrt{1.36 - 1.2\cos w}} = 2.5$$

$a = 0.6$, $w = \pi$

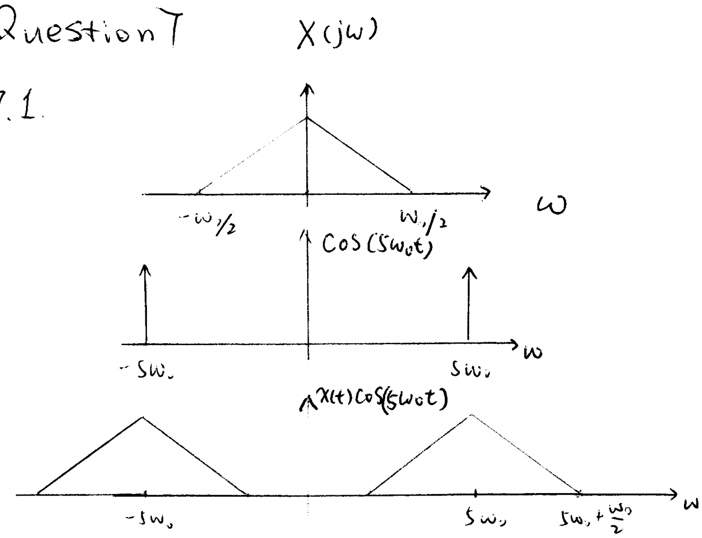
$$|H(e^{jw})| = \frac{1}{\sqrt{1.36 - 1.2\cos w}} = 0.625$$



Lowpass

Question 7

7.1



∴ The Nyquist rate for signal $x(t)\cos(5w_0 t)$

is $2\left(5w_0 + \frac{w_0}{2}\right) = 11\,w_0$

(11)

7.2

For $X(j\omega) * X(j\omega) = 0$ when $|\omega| > 15000\pi$

so we have $X(j\omega) = 0$ for $|\omega| > 7500\pi$

the Nyquist rate for this signal is $2\omega_M = 2 \times 7500\pi$

$= 15000\pi$

For $T = 10^{-4}$ s

the sampling frequency $\omega_c = \frac{2\pi}{T} = \frac{2\pi}{10^{-4}}$

$= 20000\pi$

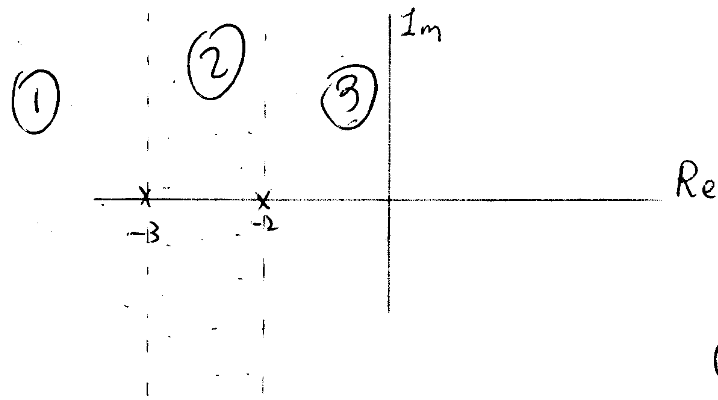
Since $\omega_c > 2\omega_M$,

$\therefore$ $x(t)$ can be recovered exactly from $x_p(t)$

Question 8

8.1. a.

$$H(s) = \frac{8}{s+2} + \frac{-15}{s+3} + 1$$



possible ROCs:

1. $Re\{s\} < -3$
2. $-3 < R\{s\} < -2$
3. $R\{s\} > -2$

b. $Re\{s\} > -2$ gives a causal system

$$h(t) = 8e^{-2t}u(t) - 15e^{-3t}u(t) + \delta(t)$$

8.2. $e(t) = x(t) - 3f(t) - 2y(t)$

(a)

$$f(t) \rightarrow \boxed{\frac{1}{s}} \rightarrow y(t) \quad \Longrightarrow \quad f(t) = \frac{dy(t)}{dt}$$

$$e(t) \rightarrow \boxed{\frac{1}{s}} \rightarrow f(t) \quad \Rightarrow \quad e(t) = \frac{df(t)}{dt} = \frac{d^2y(t)}{dt^2}$$

$$\therefore \quad \frac{d^2y(t)}{dt^2} = x(t) - 3\frac{df(t)}{dt} - 2y(t)$$

$$\therefore \quad \frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(b) $(j\omega)^2 Y(j\omega) + 3(j\omega)Y(j\omega) + 2Y(j\omega) = X(j\omega)$

$$\therefore \quad H(j\omega) = \frac{1}{(j\omega)^2 + 3j\omega + 2}$$

(c) $$H(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} = \frac{1}{j\omega+1} - \frac{1}{j\omega+2}$$

$$\therefore \quad h(t) = e^{-t}u(t) - e^{-2t}u(t)$$

(d) $$X(j\omega) = \frac{1}{j\omega+3}$$

$$Y(j\omega) = H(j\omega)\cdot X(j\omega) = \frac{\frac{1}{2}}{j\omega+3} + \frac{\frac{1}{2}}{j\omega+1} - \frac{1}{j\omega+2}$$

$$y(t) = \frac{1}{2}e^{-3t}u(t) + \frac{1}{2}e^{-t}u(t) - e^{-2t}u(t)$$

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