Université d'Ottawa · University of Ottawa

Faculté de génie
École d'ingénierie
et de technologie de l'information

Faculty of Engineering
School of Information Technology
and Engineering

ELG 3120

Signal and System Analysis

Final Exam – Fall 2005

2005-12-08 Thursday 14:00 - 17:00

Montpetit Hall Room: 202 (ELG3120A)
Montpetit Hall Room: 203 (ELG3120B)

Prof. Jianping Yao

- Time allowed: 3 Hours
- Write legibly and <u>underline</u> answers
- Initial on the top of each page
- Simple calculator is permitted: TI-30X, TI-30XA, TI-30XSLR, or TI-30XIIS, scientific, non-programmable.
- Close-book exam

Family Name:

Given name:

Student number:

Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Total	

Question 1 Continuous-Time Fourier Transform **/20**

(A) (10 points)

(a) Using the Fourier Transform integral to calculate the Fourier transform of the signal $x(t)$ shown in Fig. 1. (3 points)

(b) Plot the Fourier transform $X(j\omega)$, label the key points. (2 points)

(c) Using Fourier transform property (time shifting property) to calculate the Fourier transform of the function $y(t)$ in Fig. 2. (5 points)

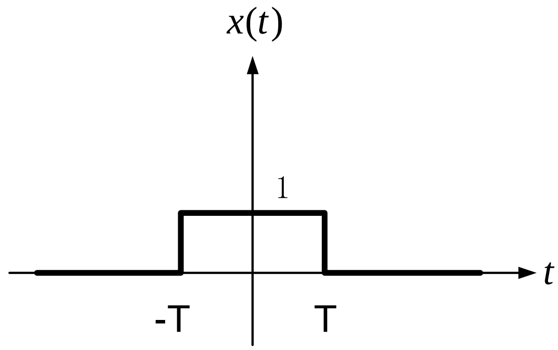


Fig. 1

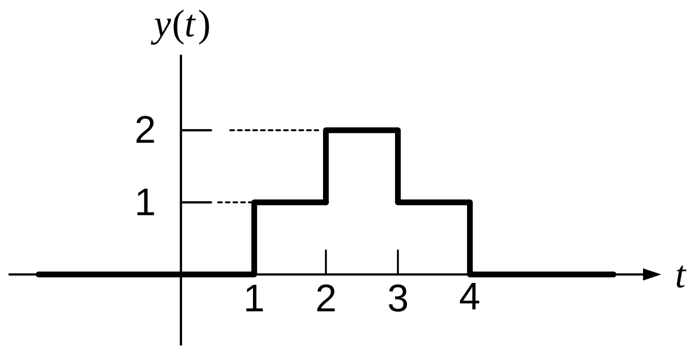


Fig. 2

(B) (10 points) Consider a system with frequency response of $H(j\omega) = \dfrac{j\omega + 2}{(j\omega + 1)(j\omega + 3)}$

(a) Find the differential equation relating the input function $x(t)$ and output $y(t)$. (3 points)

(b) Determine the impulse response $h(t)$. (3 points)

(c) If an input $x(t) = e^{-t}u(t)$ is applied to the system, what is the output. (4 points)

Question 2: Discrete-time Fourier Transform **/20**

(A) (10 points) Consider a discrete-time signal $x[n] = a^n u[n]$, $|a| < 1$,

(a) Using the Fourier transform sum to calculate its Fourier transform. (5 points)

(b) Give the expressions of its magnitude $|X(e^{j\omega})|$ and phase $\angle X(e^{j\omega})$. (5 points)

(B) (10 points) A system having the output $y[n]=\left(\frac{1}{2}\right)^n u[n]+\frac{1}{2}\left(\frac{1}{4}\right)^n u[n]$ for the input

$x[n]=\left(\frac{1}{4}\right)^n u[n]$

(a) Find the Fourier transforms of $x[n]$ and $y[n]$. (4 points)

(b) Find the frequency response $H(e^{j\omega})$. (3 points)

(c) Find the impulse response $h[n]$. (3 points)

Question 3: Modulation and demodulation **/10**

A signal $s(t)$ with a spectrum $S(j\omega) = X\left[j(\omega+\omega_0)\right] + X\left[j(\omega-\omega_0)\right]$ shown in Fig. 4 is applied to a mixer. A local oscillator signal $w(t) = \cos\omega_0 t$ is also applied to the mixer, as shown in Fig. 3.

(a) At the output of the mixer, a signal $g(t)$ is obtained. Find the Fourier transform $G(j\omega)$. (5 points)

(b) Plot $G(j\omega)$. (3 points)

(c) An ideal lowpass filter is connected after the mixer. The Transfer function of the lowpass filter is $H_{LP}(j\omega) = \begin{cases} 1, |\omega| \le \Omega \\ 0, |\omega| > \Omega \end{cases}$, plot the Fourier transform $Y(j\omega)$. (2 points)

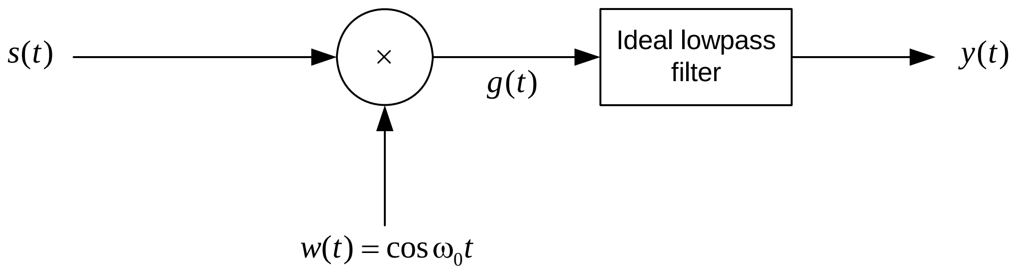


Fig. 3

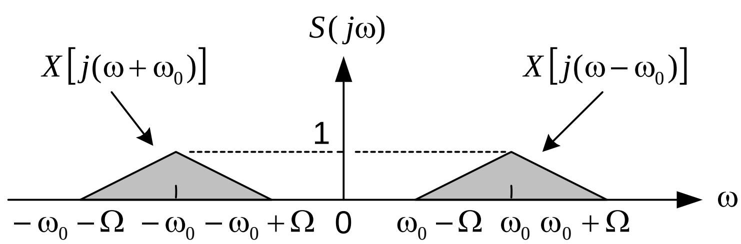


Fig. 4

Question 4 : Filtres **/15**

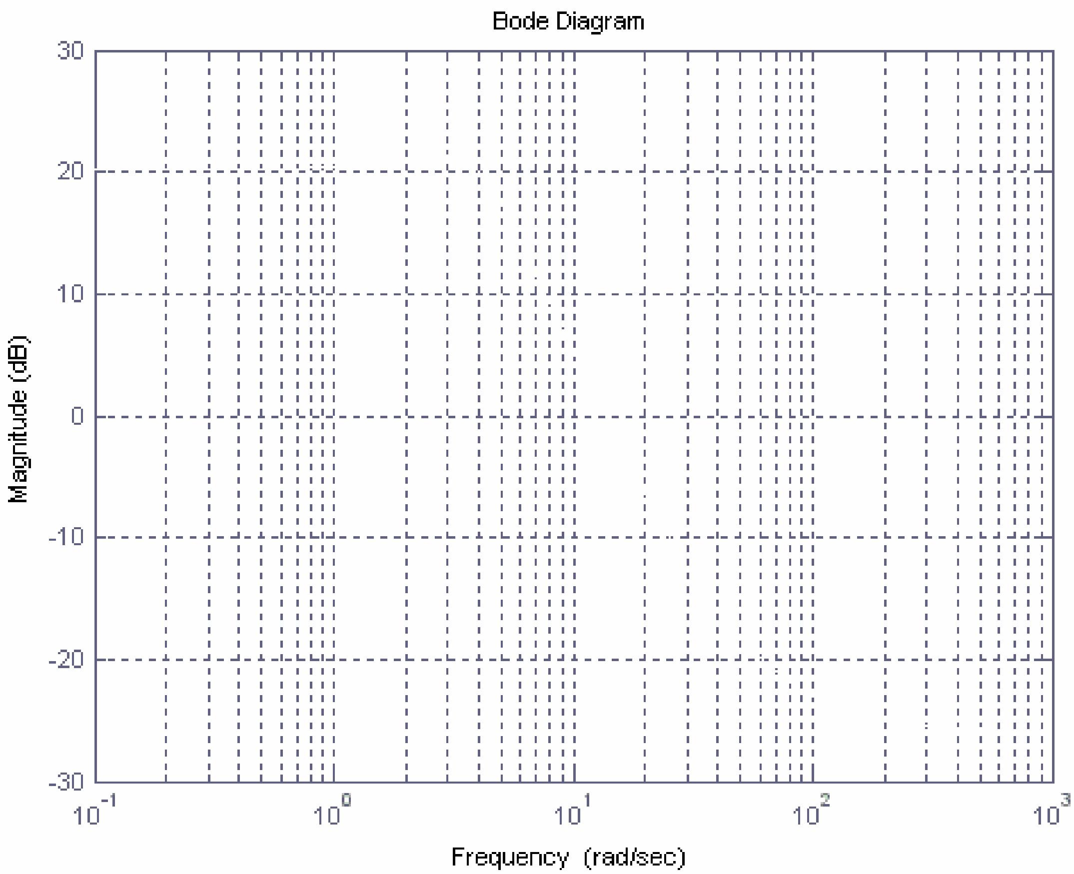
A linear time-invariant filter has system function

$$H(s) = \frac{s^2 - 120s + 3200}{20(s^2 + 4s + 16)}$$

(a) Express the frequency response $H(j\omega)$ as a product of a gain and first and second order terms in the standard form. (4 points)

(b) Give expressions for the magnitude response $|H(j\omega)|$ and the phase response $\angle H(j\omega)$. (4 points)

(c) Plot the straight-line approximation to the Bode plot of $|H(j\omega)|$ in dB on the axes below. Indicate known values on the axes and slopes of linear segments. (5 points)



(d) If a sinusoidal signal $100\cos(1000t)$ is applied as input to the above system, estimate from your Bode plot the amplitude of the sinusoidal output of the system. Give a numerical expression. (2 points)

Question 5: Sampling **/15**

(A) (10 points)

(a) A real signal $x(t)$ with Fourier transform $X(j\omega)$ shown if Fig. P5 is multiplied by the modulating function $m(t) = \cos(600\pi t)$ to yield the signal $y(t) = x(t)m(t)$ .What is the lowest possible sampling frequency ω_N at which $y(t)$ can be sampled without aliasing (i.e. the Nyquist frequency)? (5 points)

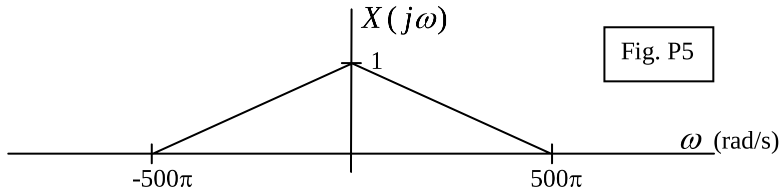


Fig. P5

(b) The signal $y(t)$ of 5(a) is sampled with an impulse train $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_N)$ to yield $z(t) = p(t)y(t)$, where $T_N = 2\pi / \omega_N$ with ω_N as found in 5(a). Sketch the Fourier transform $Z(j\omega)$ of $z(t)$ for $-2500\pi < \omega < 2500\pi$. (5 points)

(B) (5 points)
The signal $x(t) = \cos(2000\pi t) + 0.5\cos(6000\pi t)$ is sampled with an impulse train $p(t)$ with sampling frequency $\omega_S = 10000\pi$ rad/s, and the resulting signal is passed though an ideal lowpass filter with frequency response

$$H_I(j\omega) = \begin{cases} 1/5000 & |\omega| < 5000\pi \\ 0 & |\omega| > 5000\pi \end{cases}$$

to give $y(t)$. Give an exact expression for $y(t)$.

Question 6 : Laplace Transform **/20**

(a) A linear time-invariant system has impulse response $h(t) = \delta(t) + e^{-3t}u(t) + 2e^{-t}u(t)$. Is this system stable? Is it causal? Justify your answers. (5 points)

(b) Determine the system function $H(s)$ and the region of convergence for the system of 6(a). Express $H(s)$ as a rational function $N(s)/D(s)$. (5 points)

(c) Determine the system function of the inverse system $H_i(s)$ of $H(s)$ found in 6(b). (3 points)

(d) What is the region of convergence of $H_i(s)$ for the inverse system to be stable? Is the resulting inverse system causal? Determine the impulse response $h_i(t)$ of the stable inverse filter. (7 points)