Université d'Ottawa
Faculté de Génie,
École d'Ingénierie et des
Technologies de l'Information

University of Ottawa
Faculty of Engineering,
School of Information
Technology and Engineering

ELG 3120

Signal and System Analysis

Midterm

Saturday, 22 October 2005

13:00 - 14:30

Professor: Jianping Yao

- **No calculators**
- **No textbook and notes (close-book exam)**
- **Initial on the top of each page.**

Last name: ____________________

First name: ____________________

Student number: ____________________

Summations :

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{k=n_1}^{\infty} a^k = \frac{a^{n_1}}{1-a} \qquad |a| < 1$$

$$\sum_{k=0}^{n_1} a^k = \frac{1-a^{n_1+1}}{1-a} \qquad a \neq 1$$

$$\sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1} - a^{n_2+1}}{1-a} \qquad a \neq 1$$

Other:

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)$$

Convolutions :

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Continuous-time Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$a_0 = \frac{1}{T}\int_T x(t)dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)\ ,$$

$$a_k = A_k e^{j\theta_k} \qquad k \geq 1$$

Discrete-time Fourier Series:

$$x[n] = \sum_{k=<N>} a_k e^{jk(\frac{2\pi}{N})n}$$

$$a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^{t} x(t)\, dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Question 1: **/20**

(a) A continuous-time system is defined by the input-output relationship

$$y(t) = \cos(\omega t)x(t)$$

State and justify whether the system is (i) linear or non-linear; (ii) time-invariant or time-variant; (iii) memoryless or with memory; (iv) causal or non-causal; (v) stable or unstable; (vi) invertible or non-invertible.

(b) A discrete-time system is defined by the input-output relationship

$$y[n] = u(x[n]) - u(x[n]-1)$$

Where $u(\cdot)$ is the unit-step function. State and justify whether the system is (i) linear or non-linear; (ii) time-invariant or time-variant; (iii) memoryless or with memory; (iv) causal or non-causal; (v) stable or unstable; (vi) invertible or non-invertible.

Question 2: **/20**

Let $x[n]=\sin\left(\frac{2\pi n}{4}\right)u[n]$.

(a) Carefully sketch the following signals for $-10 \le n \le 10$:

(i) $x[n]$

(ii) $x[-n+3]$

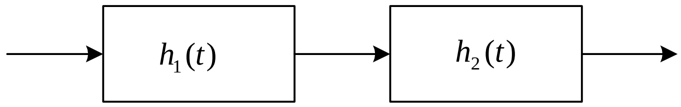
(iii) $\mathrm{Ev}\{x[n]\}$

(iv) $\mathrm{Od}\{x[n]\}$

(b) For each of the signals in (i)-(iv), state if it is periodic or non-periodic. If it is periodic, give the fundamental period.

Question 3: **/20**

Find the overall impulse response of two LTI systems connected in cascade. The impulse responses of the two subsystems are $h_1(t) = e^{-3t}u(t) + \delta(t-2)$ and $h_2(t) = u(t-3) - u(t-5)$.



Solution:

$h(t) = h_1(t) * h_2(t)$

Question 4: **/20**

Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related by the difference equation $y[n]-\frac{1}{3}y[n-1]=x[n]$

(1) Find the impulse response of the system.

(2) Determine $y[n]$ using convolution if $x[n]=u[n]-u[n-8]$.

Question 5: **/20**

(1) Calculate the Fourier series coefficients of the following continuous-time signal:

$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ -1 & 1 \le t \le 2 \end{cases}$ with a fundamental period of 2.

(2) A discrete-time periodic signal $x[n]$ is real valued and has a fundamental period $N = 5$. The nonzero Fourier series coefficients for $x[n]$ are

$a_0 = 2$, $a_2 = a_{-2}^* = 2e^{j\pi/6}$, $a_4 = a_{-4}^* = e^{j\frac{\pi}{3}}$, express $x[n]$ in the form of

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k)$$