Université d’Ottawa
Faculté de genie

École d’ingénierie et de technologie
de l’information (ÉITI)

L’Université canadienne
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University of Ottawa
Faculty of Engineering

School of Information Technology and
Engineering

ELG 3120
Signal and System Analysis

Final Exam

Time allowed: 180 minutes (3 hours)

Friday, 7 December 2007, 14h00

Professor: Jianping Yao

Family name: ______________________________

Given name: ______________________________

Student number: ___________________________

Signature: ________________________________

Closed-book exam

Calculators are not permitted

Q1	/32
Q2	/20
Q3	/8
Q4	/7
Q5	/11
Q6	/11
Q7	/11
Total	**/100**

Question 1 **/32**

a) (/8) Calculate the Fourier transform of the signal given by

$$x(t) = e^{-\frac{1}{2}t} \cos(10t + \frac{\pi}{4}) u(t)$$

b) (/8) $X(j\omega)$ is the Fourier transform of $x(t)$, $X(j\omega) = F[x(t)]$. Prove the duality property of the Fourier transform: $x(-\omega) = \frac{1}{2\pi} F[X(t)]$.

(Hint: Using the equation of the inverse Fourier transform, exchange the variables of time and frequency, and manipulate the result to get an equation of the same form as the Fourier transform.)

(c) (/7) Find the total energy (i.e. $\int_{t=-\infty}^{\infty} |x(t)|^2 dt$) of the signal: $x(t) = \frac{\sin(10t)}{\pi t}$.

(Hint: using the Parseval's relation)

d) (9/) For a causal LTI system with an input signal $x(t) = e^{-2t}u(t)$, the output signal is $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$. Determine the impulse response $h(t)$ of the system.

Question 2 **/20**

a) (/9) An LTI system is characterized by the difference equation $y[n] = x[n] - x[n-1]$,

i) (/2) If the system is causal or not? Justify your answer.

ii) (3/) Find the impulse response $h[n]$ of the system.

ii) (/4) Give the expressions of its magnitude $X(e^{j\omega})$ and phase $\angle X(e^{j\omega})$.

b) (/11) Consider a system consisting of the two cascaded LTI systems with frequency responses

$$H_1(j\omega) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}, \text{ and } H_2(j\omega) = \frac{1}{1 - e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}$$

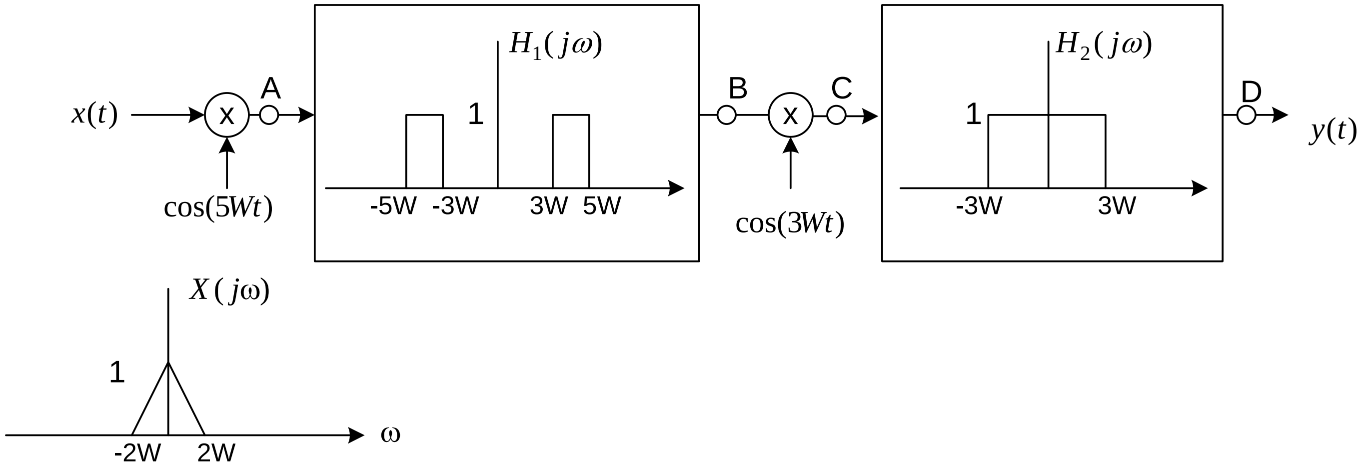
i) (/4) Find the difference equation of the overall system.

ii) (/4) Determine the impulse response of the over system.

iii) (/3) If an input signal $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$ is applied to the input of the system, what is the output response $y[n]$.

Question 3 **/8**

A system is shown in the figure below. The input signal has a Fourier transform $X(j\omega)$ shown also in the figure. Sketch the spectra at the points A, B, C and D.



Question 4 **/7**

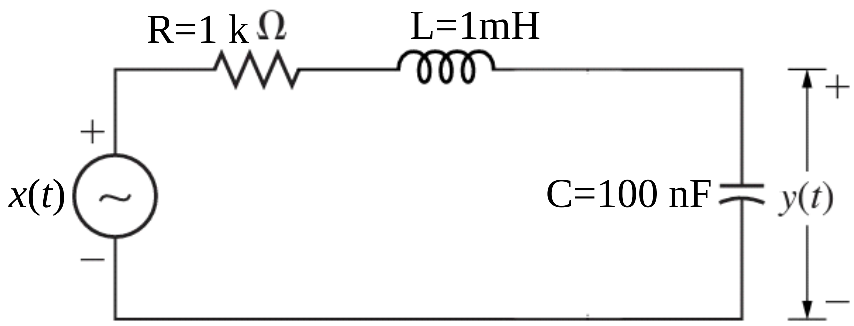
The following system can be modeled as a second-order LTI system with the standard form:

i) (/3) Write the differential equation that characterizes the system (using *R*, *L*, *C*, with no numerical values)

ii) (/2) Write the frequency response of the system (using *R*, *L*, *C*, with no numerical values)

iii)(/1) Determine if the system is under or over damped.

iv)(/1) Determine the impulse response $h(t)$ of the system contains oscillations or not (note: it is not necessary to calculate $h(t)$)



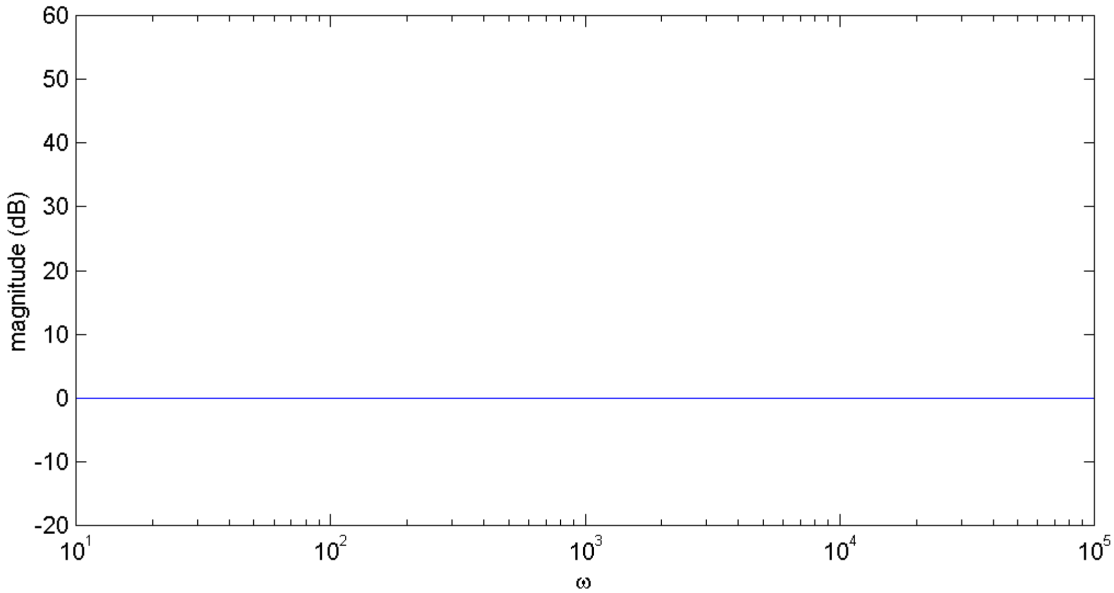
Note: $x(t)$ is the input voltage (in Volts), and $y(t)$ is the output voltage over the capacitor (in Volts).

Question 5 **/11**

An LTI system has the following transfer function:

$$H(j\omega) = \frac{10^{10}(j\omega + 100)}{(j\omega + 1000)\,((j\omega)^2 + 9000(j\omega) + 10^8)}.$$

In the figure below, draw the Bode plot of the amplitude response $|H(j\omega)|$ (in dB). Use straight line approximation. Explain how you got the plot.



Question 6 **/11**

A signal $x(t)$ has a Fourier transform given by $X(j\omega) = \omega[u(\omega) - u(\omega - \omega_0)]$.

(a) i) (/2) Plot $X(j\omega)$.

ii) (/3) Show that the signal can be sampled using an impulse train $p(t)$ with a sampling period of $T = \frac{4\pi}{3\omega_0}$, without spectral aliasing. Plot the spectrum of the sampled signal $X_P(j\omega)$.

iii) (/3) Plot the frequency response of a filter to reconstruct $X(j\omega)$ from $X_P(j\omega)$.

(b) (/3) Can the signal $s(t) = te^{-at}u(t),\ a > 0$ be perfectly reconstructed after being sampled by an impulse train? Explain why.

Question 7 **/11**

A continuous-time causal LTI system has a transfer function given below :

$$H(s) = \frac{\frac{101}{100}(s + j10)(s - j10)}{s^2 + 2s + 101}$$

(a) (/2) Find the poles and the zeros.

(b) (/2) Indicate the poles and the zeros on the s-plane. Indicate the region of convergence (ROC).

(c) (/2) Write the differential equation of the system.

(d) (/2) Prove that the gain of the system at dc is unity (or the gain is 1 at dc).

(e) (/2) If the system described by $H(s)$ is stable ? Explain.

(f) (/1) For the system descried by $H(s)$, if the Fourier transform $H(j\omega)$ exists? Explain why.

Table of formulas

Convolutions:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Continuous-time Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = a_0 + 2\sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$$

$$\omega_0 = \frac{2\pi}{T} \qquad a_k = A_k e^{j\theta_k} \quad k \geq 1$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt \qquad a_0 = \frac{1}{T}\int_T x(t)dt$$

Discrete-time Fourier Series

$$x[n] = \sum_{k=<N>} a_k e^{jk(\frac{2\pi}{N})n} \qquad a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$

Continuous-time Fourier transform and inverse Fourier transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$$

Discrete-time Fourier transform and inverse Fourier transform

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Fourier Transform of periodic signal:

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi\, a_k \delta(\omega - k\omega_0)$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} 2\pi\, a_k \delta(\omega - k\frac{2\pi}{N}) \quad \text{(period } 2\pi\text{)}$$

Continuous-time first and second order lowpass systems in standard form:

$$H(j\omega) = \frac{1}{1 + j\omega\tau}$$

$$H(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$$

Impulse-train sampling:

$$x_p(t) = x(t) \times p(t) \qquad x_d[n] = x(nT)$$

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)) \qquad \omega_s = \frac{2\pi}{T}$$

$$X_d(e^{j\omega}) = X_p(j\omega F_s) = X_p(j\omega/T)$$

Bilateral Laplace transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Unilateral Laplace transform

$$\chi^+(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

Summations:

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \qquad \sum_{k=n_1}^{\infty} a^k = \frac{a^{n_1}}{1-a} \qquad |a| < 1$$

$$\sum_{k=0}^{n_1} a^k = \frac{1-a^{n_1+1}}{1-a} \qquad \sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1} - a^{n_2+1}}{1-a} \qquad a \neq 1$$

Other:

$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathcal{R}e\{X(j\omega)\} = \mathcal{R}e\{X(-j\omega)\} \\ \mathcal{I}m\{X(j\omega)\} = -\mathcal{I}m\{X(-j\omega)\} \\ \|X(j\omega)\| = \|X(-j\omega)\| \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{R}e\{X(j\omega)\}$ $j\mathcal{I}m\{X(j\omega)\}$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

Duality: if $x(t) \overset{F.T.}{\leftrightarrow} X(j\omega)$ then $X(jt) \overset{F.T.}{\leftrightarrow} 2\pi\, x(-\omega)$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0,$ otherwise
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0,$ otherwise
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0,$ otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1, \quad a_k = 0, \ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t),$ $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

$$e^{-at}\sin(\omega_0 t)u(t) \xleftrightarrow{F.T.} \frac{\omega_0}{(j\omega + a)^2 + \omega_0^2} \qquad e^{-at}\cos(\omega_0 t)u(t) \xleftrightarrow{F.T.} \frac{j\omega + a}{(j\omega + a)^2 + \omega_0^2} \qquad a > 0,\ \omega_0 \ge 0$$

TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$ $y[n]$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ periodic with period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
5.3.4 T	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\omega}}X(e^{j\omega})$ $+\pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \mathcal{Re}\{X(e^{j\omega})\} = \mathcal{Re}\{X(e^{-j\omega})\} \\ \mathcal{Im}\{X(e^{j\omega})\} = -\mathcal{Im}\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real an even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{Ev}\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{Od}\{x[n]\}$ $[x[n]$ real$]$	$\mathcal{Re}\{X(e^{j\omega})\}$ $j\mathcal{Im}\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals	$\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$	

Duality: if $x[n] \overset{F.T.}{\leftrightarrow} X(e^{j\omega})$ then $X(e^{jt}) \overset{F.S.}{\leftrightarrow} x[-k]$.

if $x[n] \overset{F.S.}{\leftrightarrow} a_k$ then $a_n \overset{F.S.}{\leftrightarrow} \frac{1}{N}x[-k]$

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \|n\| \le N_1 \\ 0, & N_1 < \|n\| \le N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \; k \neq 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1 + 1}{N}, \; k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \|a\| < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] \begin{cases} 1, & \|n\| \le N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \|\omega\| \le W \\ 0, & W < \|\omega\| \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n+1)a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—

$$r^n \sin[\omega_0 n] u[n] \xleftrightarrow{F.T.} \frac{r \sin(\omega_0) e^{j\omega}}{e^{j2\omega} - 2r\cos(\omega_0) e^{j\omega} + r^2} \qquad r^n \cos[\omega_0 n] u[n] \xleftrightarrow{F.T.} \frac{e^{j2\omega} - r\cos(\omega_0) e^{j\omega}}{e^{j2\omega} - 2r\cos(\omega_0) e^{j\omega} + r^2}$$

$0 \le r \le 1, \quad 0 \le \omega_0 \le \pi \qquad\qquad 0 \le r \le 1, \quad 0 \le \omega_0 \le \pi$

Properties of bilateral Laplace transform

TABLE 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$	$X(s)$	R
		$x_1(t)$	$X_1(s)$	R_1
		$x_2(t)$	$X_2(s)$	R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a \rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{R}e\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to \infty} sX(s)$$

Properties of unilateral Laplace transform

If $x(t) \xleftrightarrow{ULT} \chi^+(s)$ then $\frac{dx(t)}{dt} \xleftrightarrow{ULT} s\chi^+(s) - x(0^-)$ and $\frac{d^2x(t)}{dt^2} \xleftrightarrow{ULT} s^2\chi^+(s) - sx(0^-) - x'(0^-)$.

Bilateral Laplace transform: $x(t) \xleftrightarrow{LT} X(s)$

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\mathcal{Re}\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\mathcal{Re}\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{Re}\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{Re}\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{Re}\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{Re}\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathcal{Re}\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathcal{Re}\{s\} > 0$
13	$[e^{-\alpha t}\cos \omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\mathcal{Re}\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\mathcal{Re}\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} > 0$

Unilateral Laplace transform: $x(t) \xleftrightarrow{T.L.U.} X^+(s)$

If $x(t)$ is causal ($x(t) = 0$, for $t < 0$) , the unilateral Laplace transform $X^+(s)$ of a signal is identical to $X(s)$ in the table above.