Université d'Ottawa
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de l'information (ÉITI)

uOttawa
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University of Ottawa
Faculty of Engineering

School of Information Technology and
Engineering

ELG 3120
Signals and Systems

Midterm

Time allowed: 80 minutes **Thursday, 25 October 2007, 10: 00 AM**

Professor: Jianping Yao **Page 1 of 6**

Family name: _____ **/25**

Given name: _____

Student number _____

Signature _____

Q1 /6	
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Close Book Exam

No calculators are permitted

Table of Formulas

Convolutions:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Continuous-time Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$a_0 = \frac{1}{T}\int_T x(t)dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k),$$

$$a_k = A_k e^{j\theta_k} \quad k \geq 1$$

Discrete-time Fourier Series:

$$x[n] = \sum_{k=<N>} a_k e^{jk(\frac{2\pi}{N})n}$$

$$a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$

Summations:

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{k=n_1}^{\infty} a^k = \frac{a^{n_1}}{1-a} \qquad |a| < 1$$

$$\sum_{k=0}^{n_1} a^k = \frac{1-a^{n_1+1}}{1-a} \qquad a \neq 1$$

$$\sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1}-a^{n_2+1}}{1-a} \qquad a \neq 1$$

Others:

$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Question 1 **/6**

For an input signal $x(t) = \delta(t) + e^{-t}u(t)$ and a LTI system with an impulse response $h(t) = e^{t}u(-t)$, find the output $y(t)$.

Question 2 **/2+3**

a) Find the impulse response of a discrete-time causal LTI system described by a difference equation given by

$$y[n] = \frac{5}{6} y[n-1] + x[n]$$

Note: find a closed form solution and do not use transforms.

b) For a causal LTI system that is described by the following differential equation

$3\frac{dy(t)}{dy} - 2y(t) = x(t)$, find its impulse response $h(t)$.

Note: do not use transforms.

Question 3 **/7**

For a system we observe that

- For an input $x(t) = e^{-t}u(t-1)$ the output is $y(t) = 5e^{-2t}u(t-5)$
- For another input $x(t) = 2e^{-(t-1)}u(t-2) + 3e^{-(t-4)}u(t-5)$ we have the output $y(t) = 10e^{-2(t-1)}u(t-6) + 15e^{-2(t-4)}u(t-9)$

Based on the observation
a) Determine if the system is LTI or not. Justify your answer.
b) If the system is causal or not? Justify.
c) If the system is stable or not? Justify.

Question 4 **/7**

a) Find the Fourier series coefficients a_k of the following periodic signal with period $T = 1$)

$x(t) = e^{-t} \quad 0 \le t < 1$

b) Find the expression for the amplitude of the coefficients, that is, $|a_k|$.

c) Plot approximately the function $|a_k|$ with respect to *k* and determine the maximum value of the harmonic component that is located at low frequency or high frequency.