Université d'Ottawa · University of Ottawa

Faculté de génie
École d'ingénierie
et de technologie de l'information

Faculty of Engineering
School of Information Technology
and Engineering

ELG 3120

Signal and System Analysis

Final Exam – Fall 2004

Tuesday 21 December 2004 9:30 - 12:30

ART 026 (ELG3120A), ART 257 (ELG3120B)

Prof. Jianping Yao

Time allowed: 3 Hours

No calculators are permitted
Close-book exam

Family Name:

Given name:

Student number:

Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Total	

Question 1 Continuous Fourier Transform **/10**

(A) (5 points) Consider that signal $x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & elsewhere \end{cases}$,

(a) determine its Fourier transform.

(b) using Fourier Transform properties, find the Fourier transform of the function $x_1(t)$ shown in Figure 1.

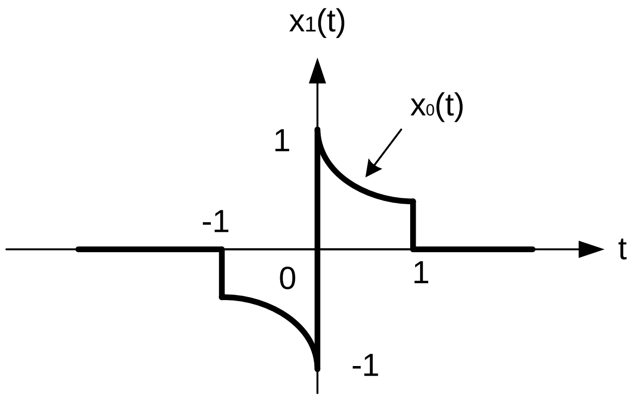


Figure 1

(B) (5 points) A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 2}{(j\omega)^2 + 4j\omega + 3}$$

(a) Determine the impulse response $h(t)$ of the system.

(b) If an input signal $x(t) = te^{-2t}u(t)$ is applied to the input of the system, find the output signal $y(t)$.

Question 2: Discrete-time Fourier Transform **/5**

Consider a discrete-time system with unit impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{4}\right)^n u[n]$

(a) Find its Fourier transform
(b) Determine a linear constant-coefficient difference equation relating the input and output of the system.

Question 3: Modulation and demodulation **/4**

Let $s(t)$ be a signal whose spectrum $S(j\omega)$ is depicted in the Figure 2 below. Also consider the signal $p(t) = \cos\omega_0 t$,

(a) Find and plot the spectrum of $r(t) = s(t)p(t)$ (that is, $R(j\omega)$).

(b) The signal $r(t) = s(t)p(t)$ is a modulated signal generated at a transmitter. To recover the original signal $s(t)$ at a receiver, a demodulation operation is required. Part of the demodulation operation is to do the multiplication of the two signals of $r(t)$ and $p(t)$ where $p(t) = \cos\omega_0 t$ is generated by a local oscillator. Find and plot the spectrum of $g(t) = p(t)r(t)$.

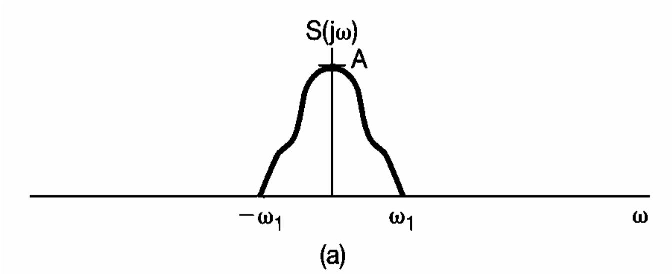
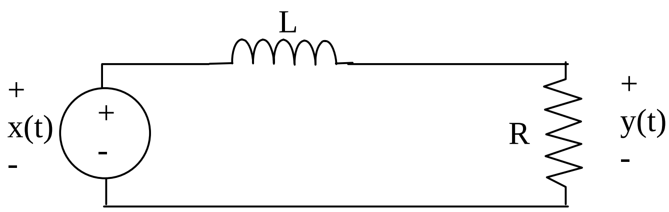


Figure 2

Question 4 First- and second-order systems **/14**

A) (6 points) The following circuit with L = 1 mH et R = 1 kΩ



Find:

A.1) The Frequency response $H(j\omega)$ and the time constant τ
A.2) The impulse response $h(t)$
A.3) The unit-step response $s(t)$
A.4) The Bode plot (amplitude et phase)

B) (3 points) A system is characterized by the following differential equation:

$$\frac{d^2 y(t)}{dt^2} + 3.5\frac{dy(t)}{dt} + 1.5y(t) = 10x(t)$$

Determine if the system is over-damped or under-damped? (note: $\sqrt{2} \approx 1.4$, $\sqrt{3} \approx 1.7$)

C) (5 points) A first-order discrete-time system:

$y[n] - ay[n-1] = x[n]$

To be a stable system, it is required that $|a| < 1$.

C.1) Determine for which value of a, the system is a low-pass filter and for which value of a the system is a high-pass filter? To verify, evaluate the gain $\left|H(e^{j\omega})\right|$ of the filter at $\omega = 0$ and at $\omega = \pi$ for $a = 0.9$ and $a = -0.9$.

C.2) For $a = 0.9$ and for $a = -0.9$, find the impulse response $h[n]$. For which values of a, the impulse response has an alternation versus *n*? For which values of a, the impulse response does not have an alternation versus *n*?

Question 5 Sampling **/7**

A) (5 points) A signal $x(t)$ is multiplied by a impulse train $p(t)$ of period T :

$$x_p(t) = x(t) \cdot p(t)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

A.1) Prove that the Fourier transform of $p(t)$ is $P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$ $(\omega_s = \frac{2\pi}{T})$

A.2) With $P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$, prove that $X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$

B) (2 points) A complex signal $x(t)$ with a Fourier transform of $X(j\omega)$ is zero everywhere except for the interval $-5 < \omega < 2$. Using $X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$, determine the minimum sampling frequency ω_s to ensure a reconstruction of the original signal $x(t)$ without losing information.

Question 6 Laplace Transform **/5**

The Laplace transform of an LTI system is shown below

$$H(s) = \frac{1}{(s+3)(s+1)(s-2)}$$

a) Find the 4 possible ROCs (regions of convergence) of the system.
b) Find the ROC that corresponds to a causal system.
c) For the ROC of b) (causal system), find the impulse response $h(t)$ of the system and prove that the system is causal.
d) Find the ROC that corresponds to stable system.
e) For the ROC de d) (stable system), Find the impulse response $h(t)$ of the system.
f) Determine if $H(s)$ can have a system that is both causal and stable? Explain why if your answer is yes, give the corresponding ROC.