



ELG 4571

Systèmes de télécommunications

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Télécommunications

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Détection cohérente

ASK, BPSK et FSK

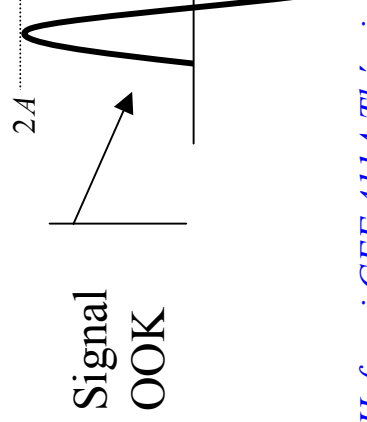
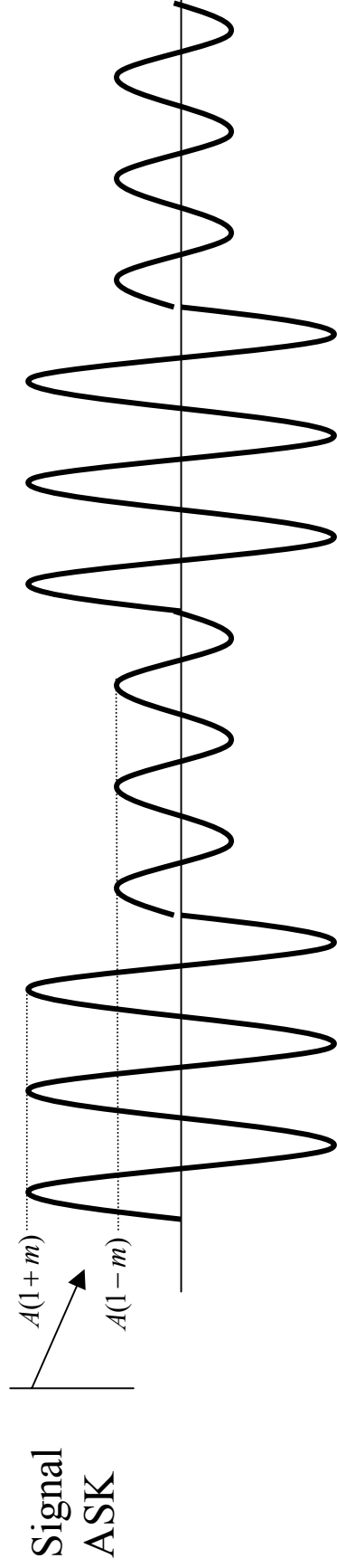
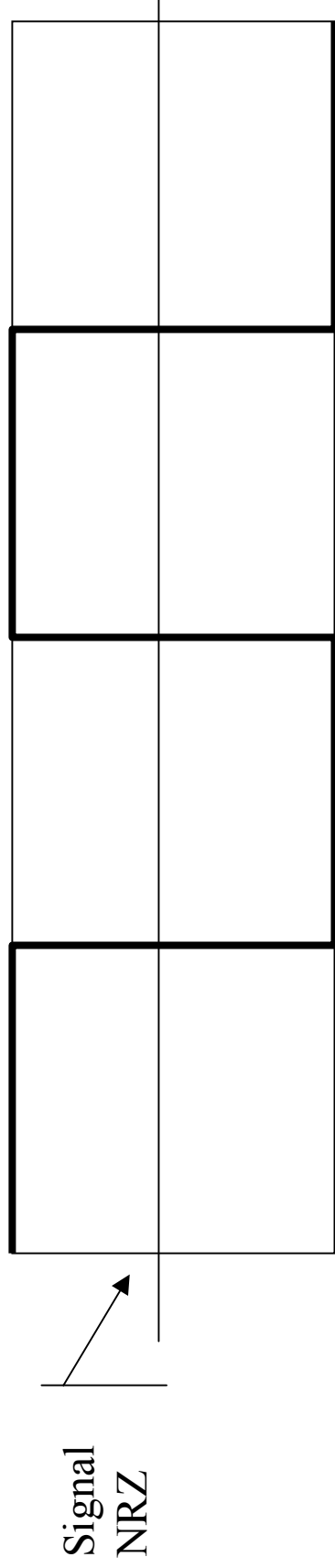
Modulation par déplacement d'amplitude (MDA) Amplitude Shift Keying (ASK)

✿ Il s'agit du cas particulier de la modulation AM ou le signal modulant $s(t)$ est une forme d'onde PCM en bande de base. Si par exemple on choisit le format NRZ-polaire, le signal AM s'exprimera dans ce cas comme suit:

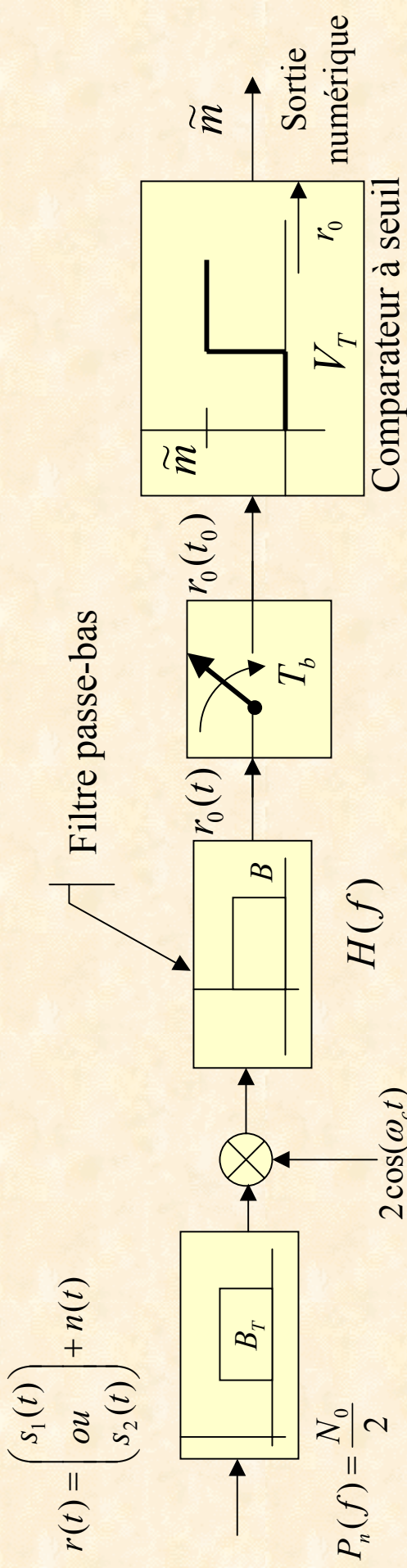
$$s_m(t) = [A \pm V] \cos 2\pi f_c t = A \left[1 \pm \frac{V}{A} \right] \cos 2\pi f_c t = [1 \pm m] \cos 2\pi f_c t$$

- Pour la transmission du bit 1 la sinusoïde d'amplitude $A(1+m)$ est utilisée.
- Pour la transmission du bit 0 la sinusoïde d'amplitude $A(1-m)$ est utilisée.
- Pour $m=1$ l'amplitude varie de 0 à $2A$. Il s'agit du cas particulier de la modulation ASK appelé OOK (On-Off Keying). Plus loin nous constaterons que la valeur $m=1$ offre les meilleures performances

ASK



Détection cohérente par filtre passe-bas d'un signal ASK



$$\begin{cases} s_1(t) = A \cos(\omega_c t), & 0 < t \leq T & \text{pour le bit 1} \\ s_2(t) = 0 & , & 0 < t \leq T & \text{pour le bit 0} \end{cases}$$

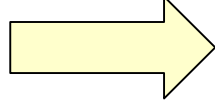
$$n(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t$$

$$r_0(t) = \begin{cases} A, & 0 < t \leq T & \text{pour le bit 1} \\ 0, & 0 < t \leq T & \text{pour le bit 0} \end{cases} + x(t)$$

Détection cohérente par filtre passe-bas d'un signal ASK

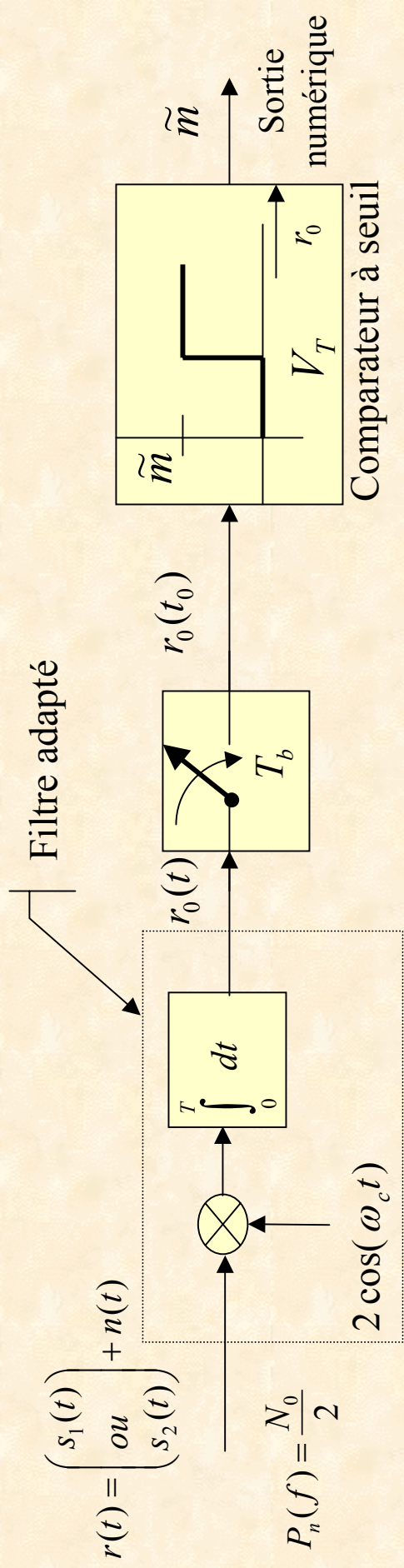
$$\begin{cases} s_{01}(t_0) \approx +A, & 0 < t \leq T & \text{pour le bit 1} \\ s_{02}(t_0) \approx 0, & 0 < t \leq T & \text{pour le bit 0} \end{cases} \quad \Rightarrow \quad V_T = \frac{s_{01} + s_{02}}{2} = \frac{1}{2}A$$

$$\left\{ \begin{aligned} P_e &= Q \left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}} \right) \\ \sigma_0^2 &= \overline{x^2(t)} = \overline{n_0^2(t)} = 2 \left(\frac{N_0}{2} \right) 2B = 2N_0B \end{aligned} \right.$$



$$P_e = Q \left(\sqrt{\frac{A^2}{8N_0B}} \right)$$

Détection cohérente par filtre adapté d'un signal ASK



$$E_d = \int_0^T [A \cos(\omega_c t) - 0]^2 dt = \frac{A^2 T}{2}$$

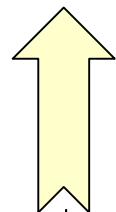
$$E_b = \frac{E_1 + E_0}{2} = \frac{A^2 T / 2 + 0}{2} = \frac{A^2 T}{4}$$

$$V_T = \frac{s_{01} + s_{02}}{2} = \frac{1}{2} s_{01} = \frac{1}{2} \left[\int_0^T 2A \cos^2(\omega_c t) dt \right] = \frac{AT}{2}$$

$$P_e = Q \left(\sqrt{\frac{A^2 T}{4N_0}} \right) = Q \left(\sqrt{\frac{E_b}{N_0}} \right)$$

$$= P_e(\text{NRZ-unipolaire})$$

Modulation par déplacement de phase (MDP) Phase Shift Keying (PSK)

$$\left\{ \begin{array}{l} s_0 = A \cos [2\pi f_c t - \Delta\varphi] \\ s_1 = A \cos [2\pi f_c t + \Delta\varphi] \\ s_i = A \cos [2\pi f_c t + d_i(t)\Delta\varphi] \end{array} \right\}$$


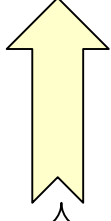
$$s_i = A \cos (2\pi f_c t) \cos [d_i(t)\Delta\varphi] - A \sin (2\pi f_c t) \sin [d_i(t)\Delta\varphi]$$

$$s_i = A \cos (2\pi f_c t) \cos [\Delta\varphi] - A d_i(t) \sin (2\pi f_c t) \sin [\Delta\varphi]$$

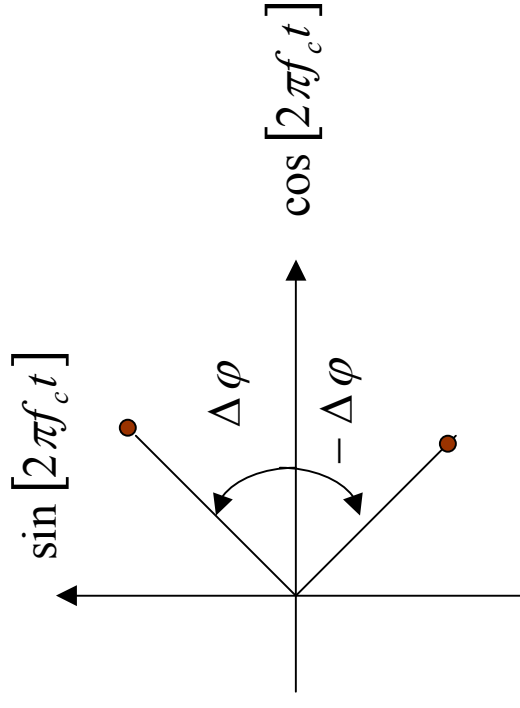
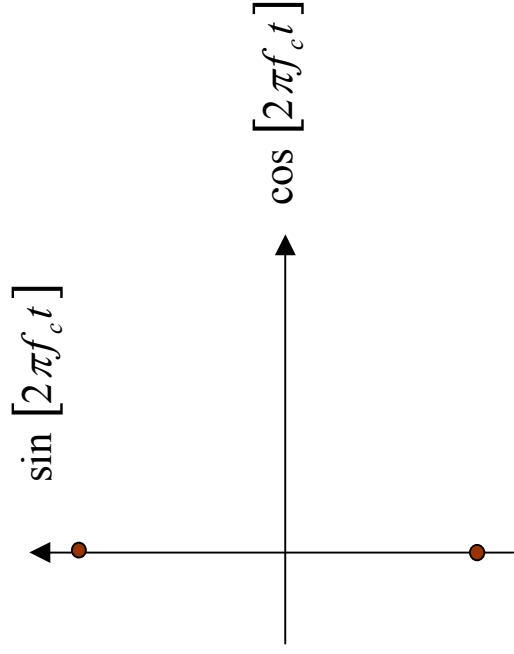
$$\Delta\varphi = \pi / 2 \rightarrow s_i = -A d_i(t) \sin (2\pi f_c t)$$

PSK

$$\left\{ \begin{array}{l} s_{PSK}(t) = \Re \{ g(t) e^{j\omega_c t} \} \\ g(t) = A e^{j\theta(t)} \\ \theta(t) = D_p m(t) \end{array} \right\}$$

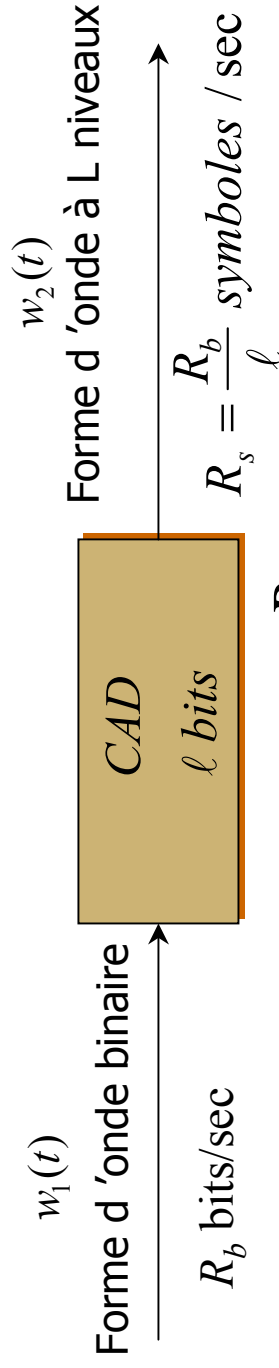


$$\left\{ \begin{array}{l} s_0(t) = \Re \{ A e^{-j\Delta\varphi} e^{j\omega_c t} \} \\ s_1(t) = \Re \{ A e^{+j\Delta\varphi} e^{j\omega_c t} \} \\ s_i(t) = \Re \{ g_i(t) e^{j\omega_c t} \} \\ g_i(t) = A e^{j d_i(t) \Delta\varphi} \end{array} \right\}$$



Signaux multiniveaux

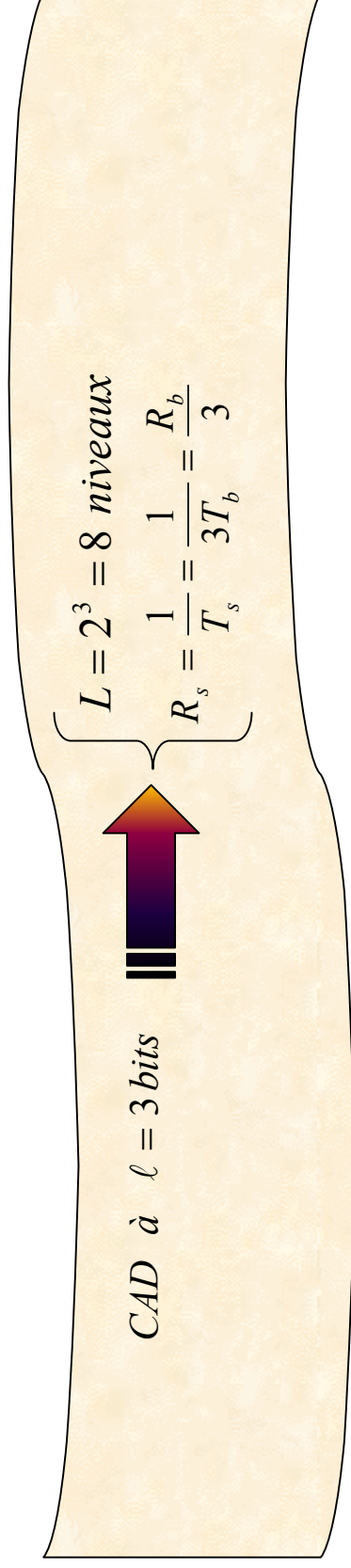
✿ Signaux multiniveaux



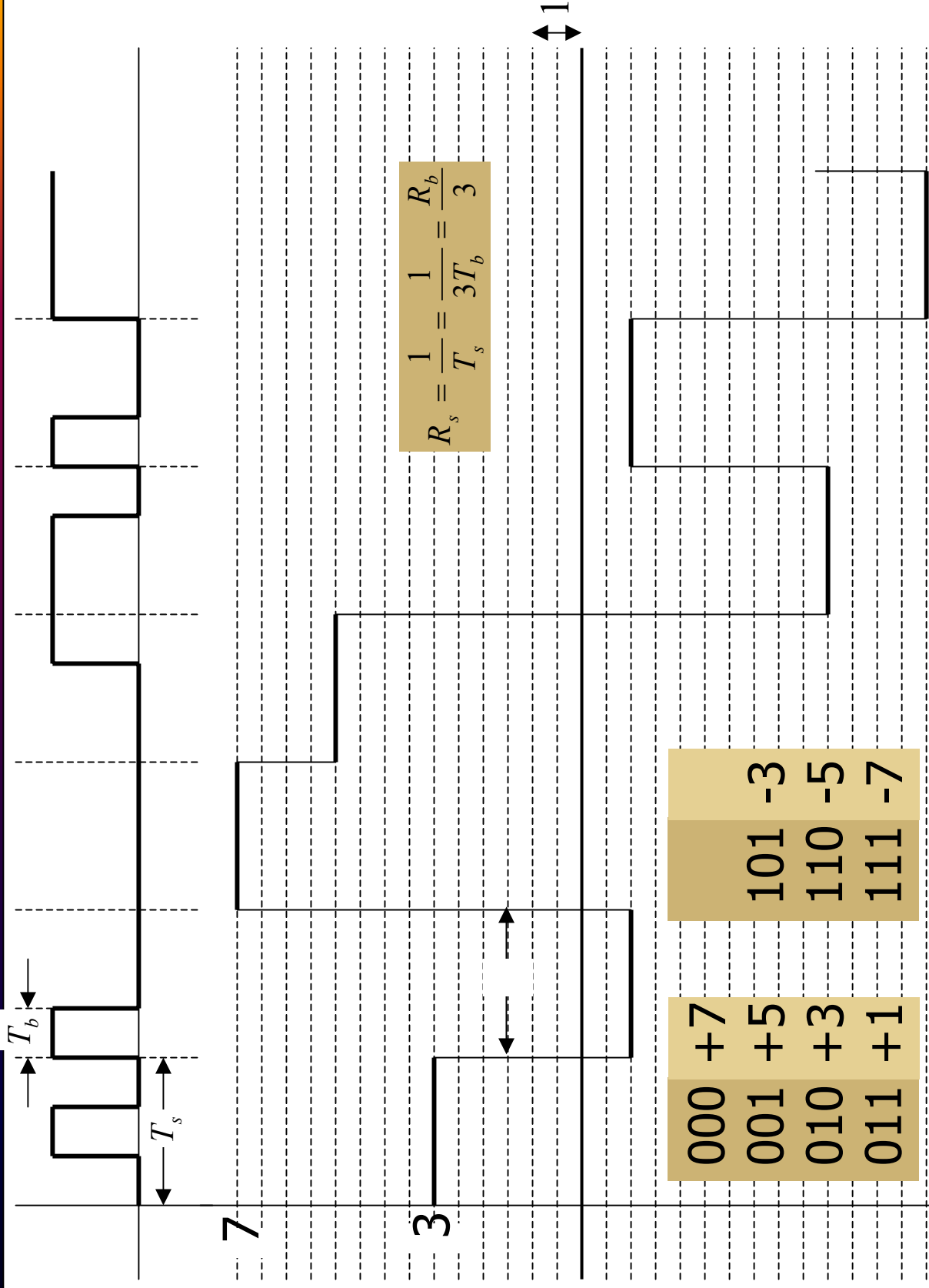
$$L = 2^l \rightarrow \log_2 L = l \rightarrow R_s = \frac{R_b}{L} \Leftrightarrow R_s \log_2 L = R_b$$

✿ Les signaux multiniveaux permettent une transmission avec une largeur de bande inférieure à celle des signaux binaires.

Exemple:



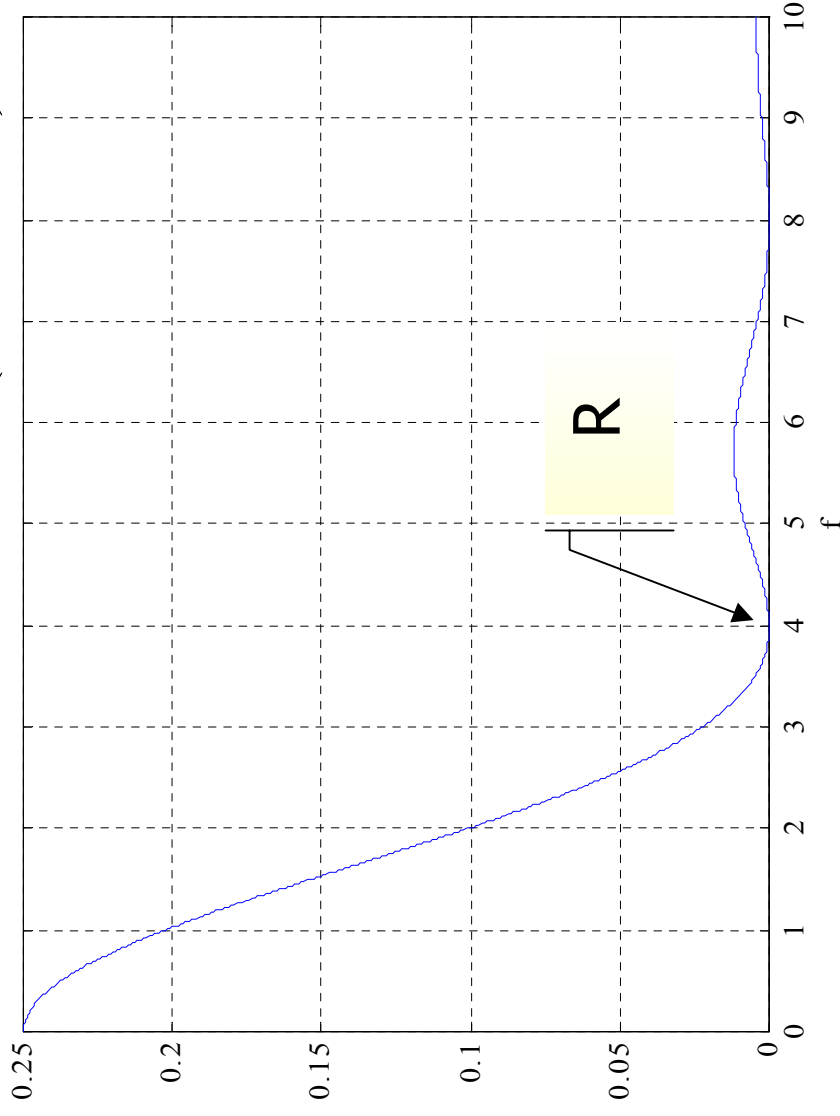
Signaux multiniveaux



Densité spectrale de puissance du code NRZ polaire

✿ PSD du code NRZ polaire: $A=1$; $T_b=0.25$

$$P_{NRZ \text{ polaire}}(f) = A^2 T_b \left(\frac{\sin \pi f T_b}{\pi f T_b} \right)^2$$



✿ Cas général

ℓ bits $L = 2^\ell$ niveaux

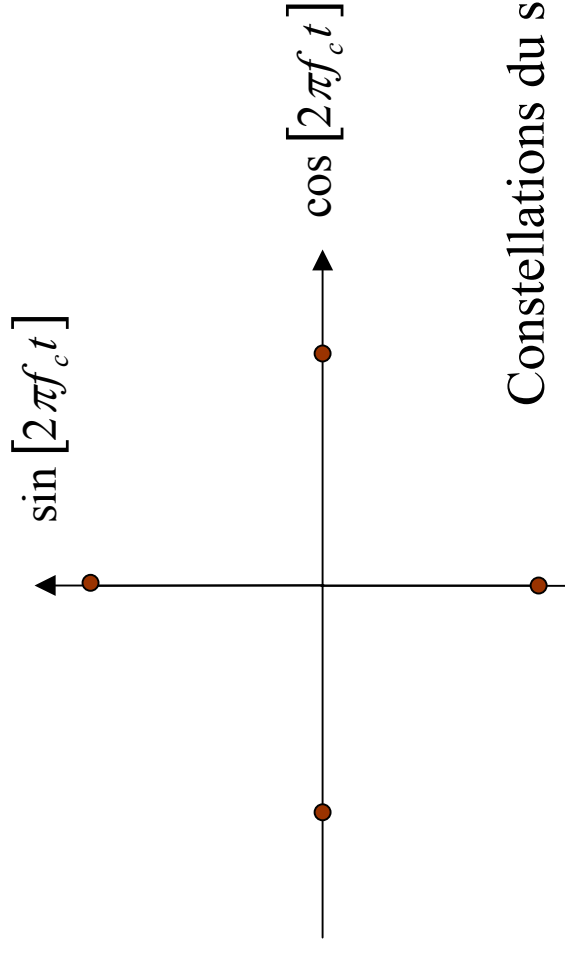
$$P_{NRZ \text{ multiniveaux}}(f) = k \left(\frac{\sin \ell \pi f T_b}{\ell \pi f T_b} \right)^2$$

$$B_{1^{\text{er}} \text{ zéros}} = \frac{1}{\ell T_b} = \frac{R}{\ell}$$

PSK 4-aires (QPSK)

$g(t) = Ae^{j\theta(t)}$ Contiendra 4 points

+1 +1	-3	$+\cos 2\pi f_c t = \cos(2\pi f_c t + 0)$
+1 -1	-1	$-\sin 2\pi f_c t = \cos(2\pi f_c t + \pi/2)$
-1 -1	+1	$-\cos 2\pi f_c t = \cos(2\pi f_c t + \pi)$
-1 +1	+3	$+\sin 2\pi f_c t = \cos(2\pi f_c t + 3\pi/2)$

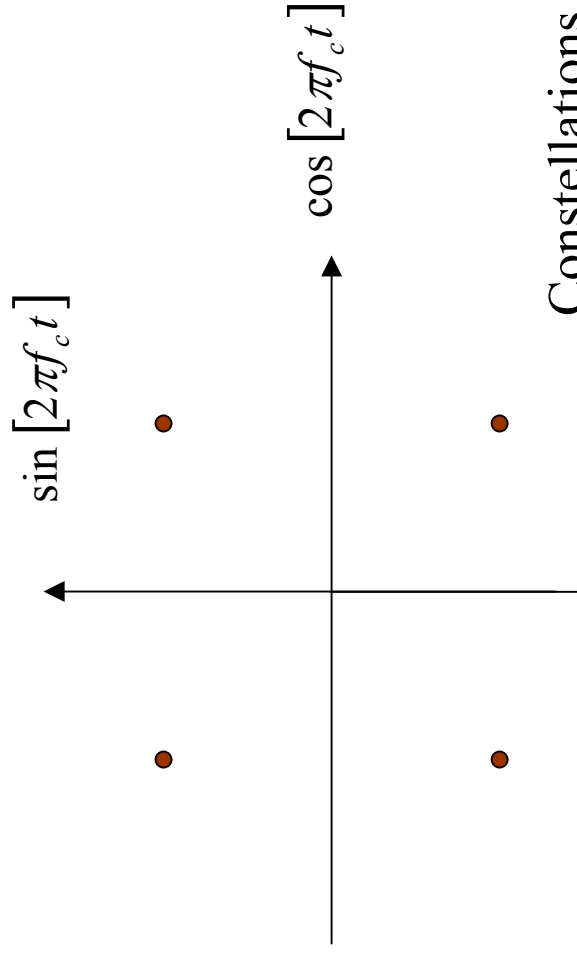


Constellations du signal QPSK

QPSK

$$g(t) = Ae^{j\theta(t)}$$

+1 +1	-3	$+\cos 2\pi f_c t = \cos(2\pi f_c t + \pi/4)$
+1 -1	-1	$-\sin 2\pi f_c t = \cos(2\pi f_c t + 3\pi/4)$
-1 -1	+1	$-\cos 2\pi f_c t = \cos(2\pi f_c t + 5\pi/4)$
-1 +1	+3	$+\sin 2\pi f_c t = \cos(2\pi f_c t + 7\pi/4)$

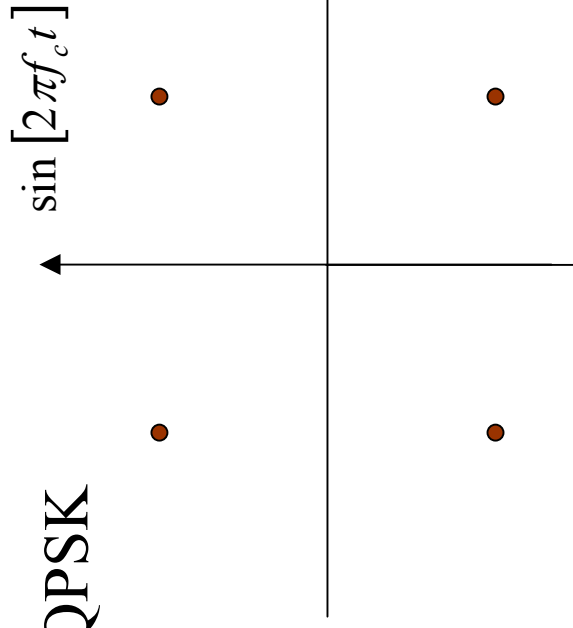


Constellations du signal QPSK

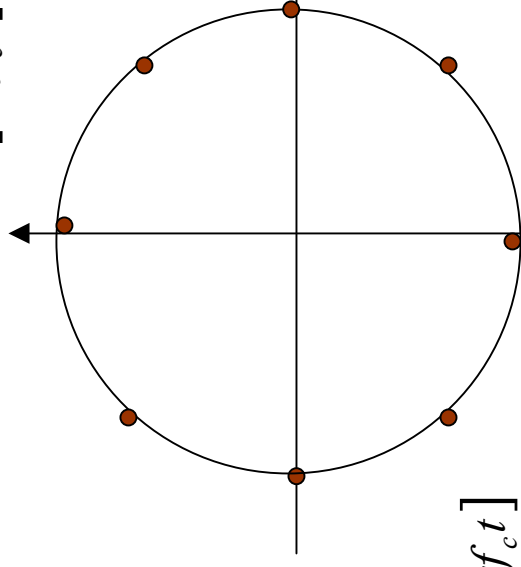
PSK M-aires

$$g(t) = Ae^{j\theta(t)}$$

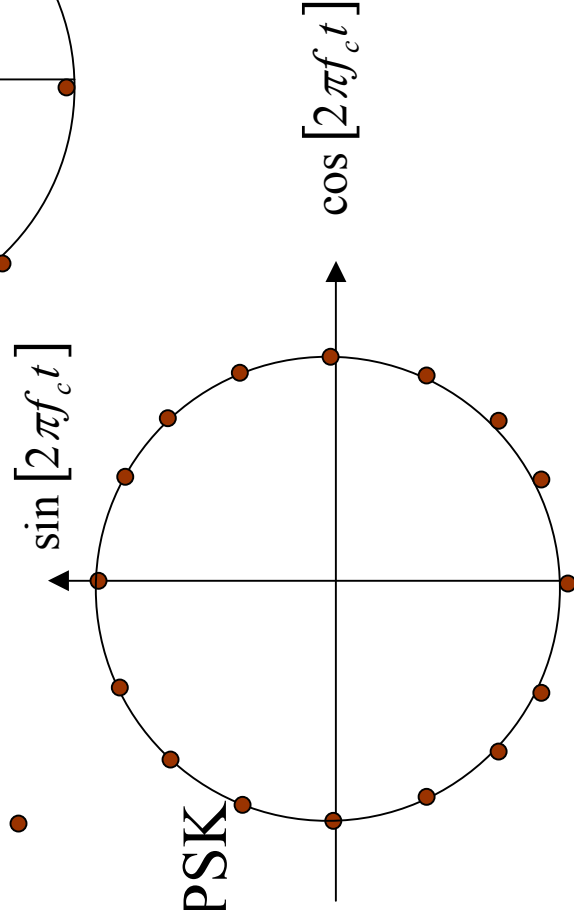
QPSK



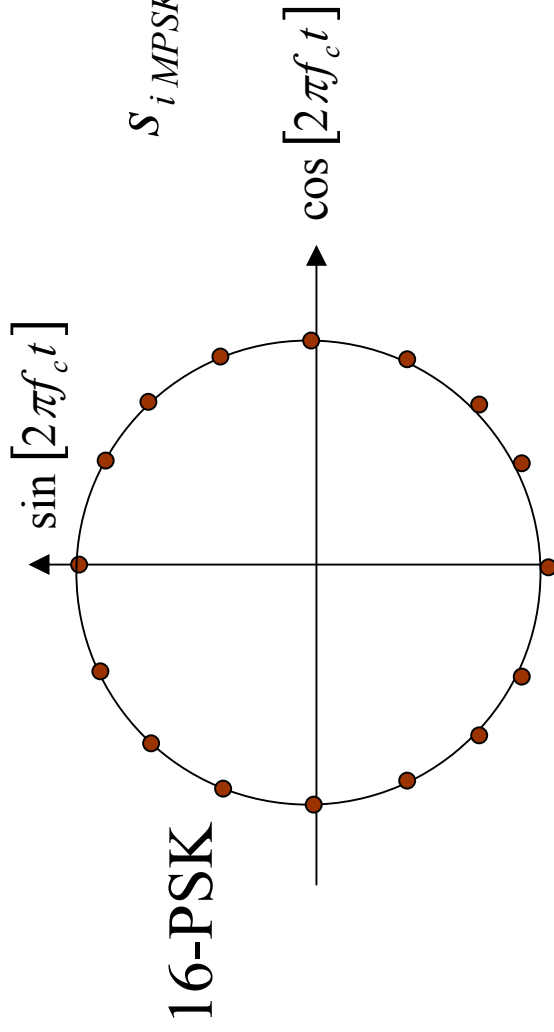
8-PSK



16-PSK

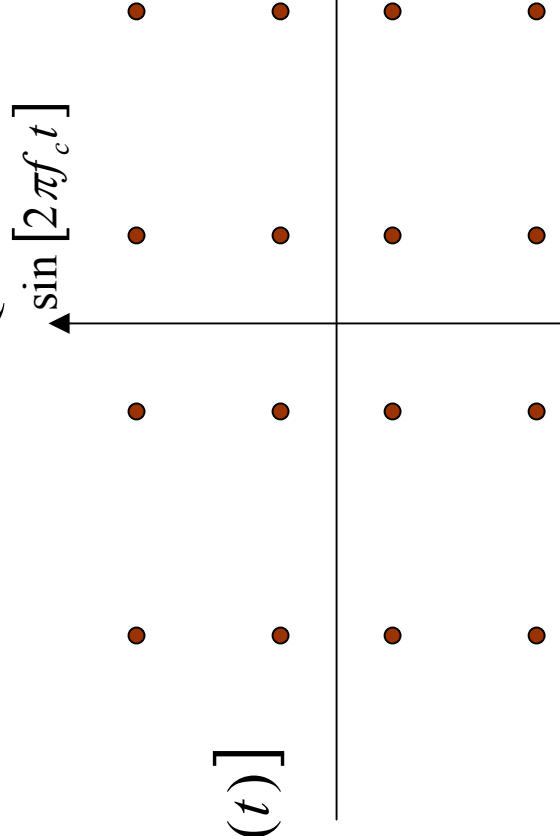


16-PSK versus 16-QAM



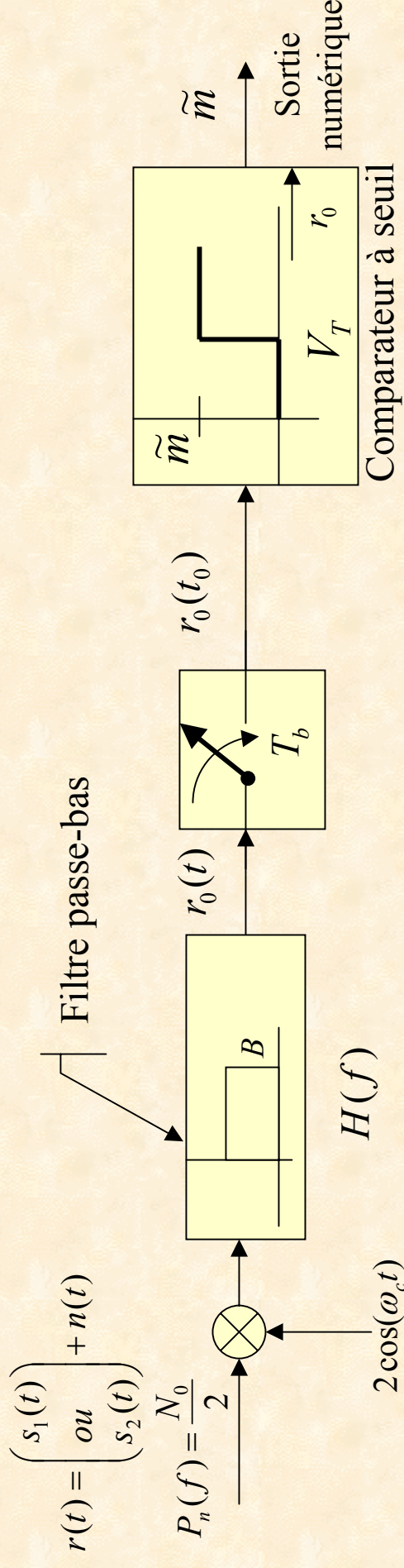
$$S_{iMPSK}(t) = A \cos [2\pi f_c t + \theta_i(t)]$$

16-QAM



$$S_{iQAM}(t) = A_i \cos [2\pi f_c t + \theta_i(t)]$$

Détection cohérente par filtre passe-bas d'un signal BPSK



$$\begin{cases} s_1(t) = A \cos(\omega_c t), & 0 < t \leq T \quad \text{pour le bit 1} \\ s_2(t) = -A \cos(\omega_c t) & , \quad 0 < t \leq T \quad \text{pour le bit 0} \end{cases}$$

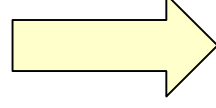
$$n(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t$$

$$r_0(t) = \begin{cases} A, & 0 < t \leq T \quad \text{pour le bit 1} \\ -A, & 0 < t \leq T \quad \text{pour le bit 0} \end{cases} + x(t)$$

Détection cohérente par filtre passe-bas d'un signal BPSK

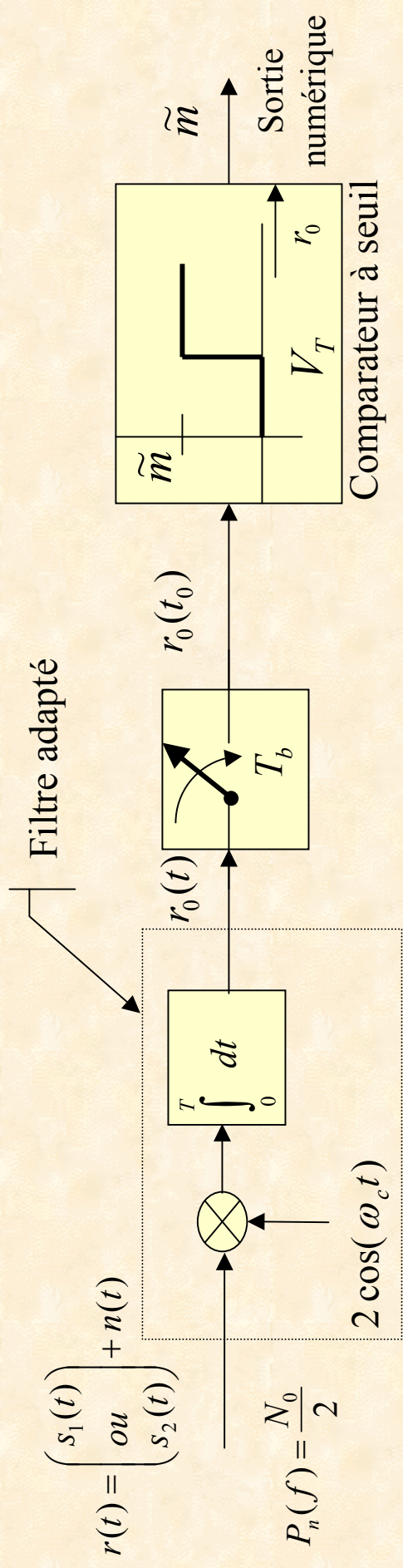
$$\begin{cases} s_{01}(t_0) \approx +A, & 0 < t \leq T & \text{pour le bit 1} \\ s_{02}(t_0) \approx -A, & 0 < t \leq T & \text{pour le bit 0} \end{cases} \quad \Rightarrow \quad V_T = \frac{s_{01} + s_{02}}{2} = 0$$

$$\begin{cases} P_e = Q \left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}} \right) \\ \sigma_0^2 = \overline{x^2(t)} = \overline{n_0^2(t)} = 2 \left(\frac{N_0}{2} \right) 2B = 2N_0B \end{cases}$$



$$P_e = Q \left(\sqrt{\frac{A^2}{2N_0B}} \right)$$

Détection cohérente par filtre adapté d'un signal BPSK



$$E_d = \int_0^T [2A \cos(\omega_c t)]^2 dt = 2A^2T$$

$$E_b = \frac{E_1 + E_0}{2} = \frac{A^2T/2 + A^2T/2}{2} = \frac{A^2T}{2}$$

$$P_e = Q\left(\sqrt{\frac{A^2T}{N_0}}\right) = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right)$$

Modulation par déplacement de fréquence (MDF) Frequency Shift Keying (FSK)

$$\lambda_{FM}(t) = A \cos [\Theta(t)]$$

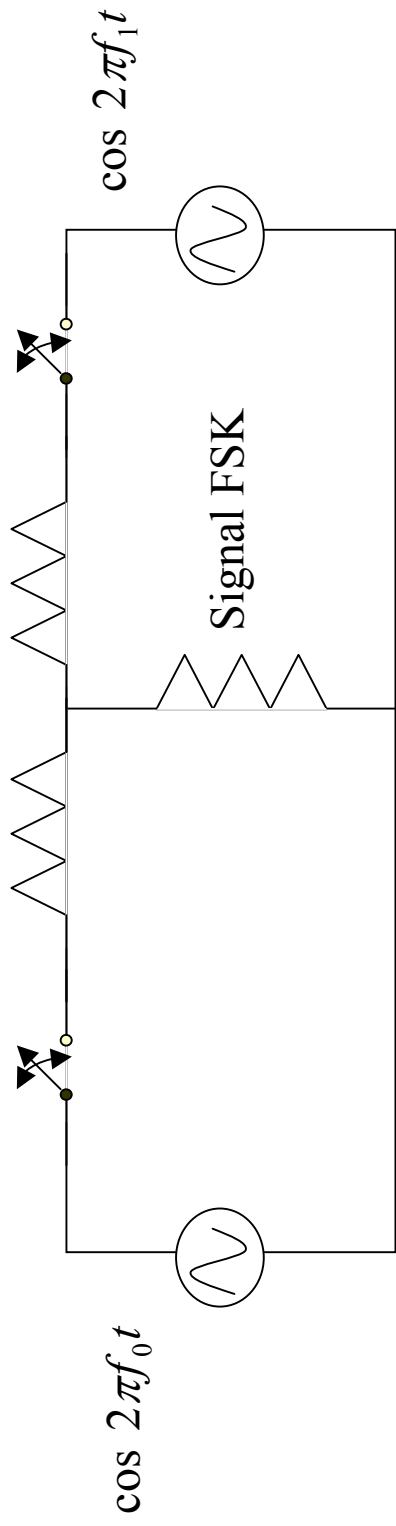
$$\Theta(t) = 2\pi f_c t + D_f \int s(t) dt$$

$$\left\{ \begin{array}{l} s_{FSK}(t) = A \cos [2\pi f(t)] \\ f(t) = f_c + \frac{D_f}{2\pi} m(t) \end{array} \right\} \xrightarrow{\text{yellow arrow}}$$

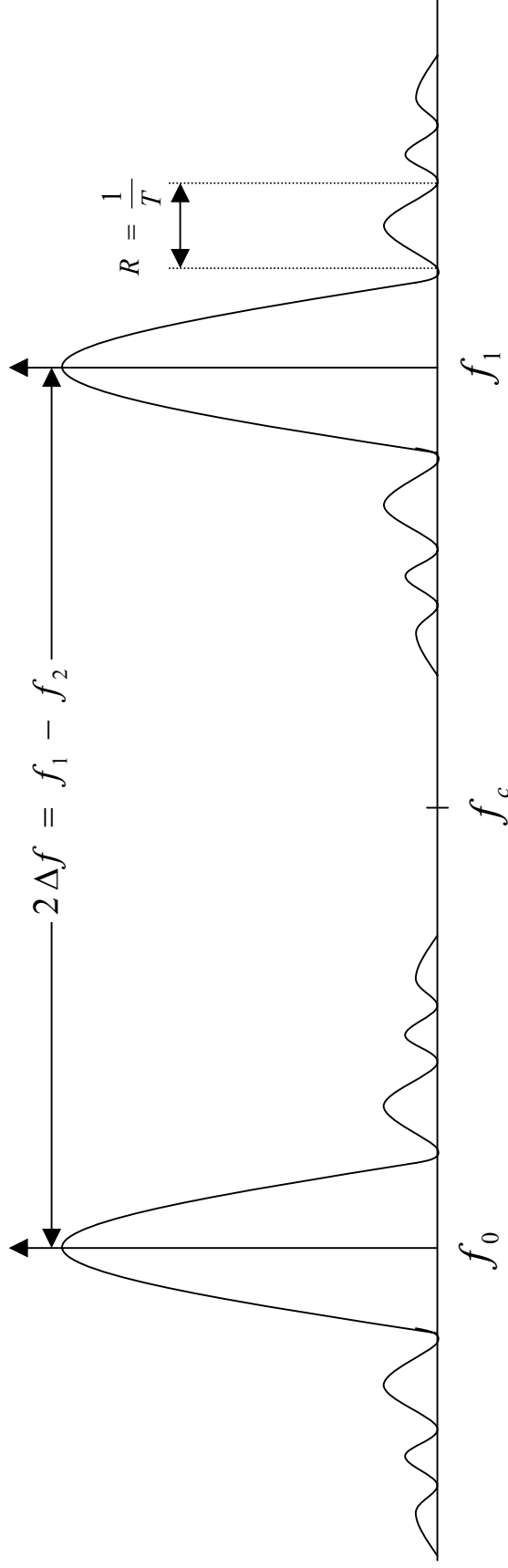
$$\left\{ \begin{array}{l} s_0 = A \cos [2\pi f_0 t] \\ s_1 = A \cos [2\pi f_1 t] \\ f_0(t) = f_c - \Delta f \\ f_1(t) = f_c + \Delta f \end{array} \right\}$$

$$\left\{ \begin{array}{l} s_{FSK}(t) = \Re e \{ g(t) e^{j\omega_c t} \} \\ g(t) = A e^{j\theta(t)} \\ \theta(t) = D_f \int s(t) dt \end{array} \right\}$$

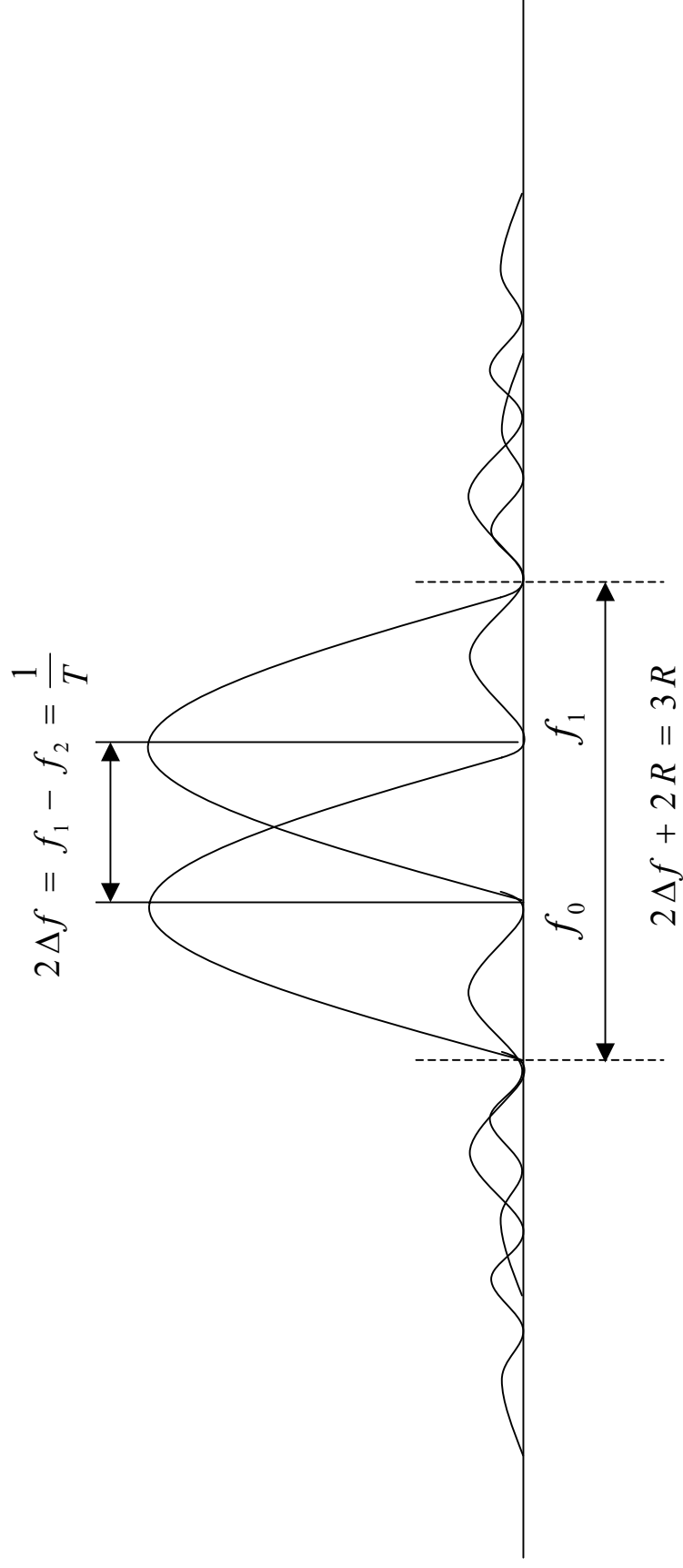
FSK



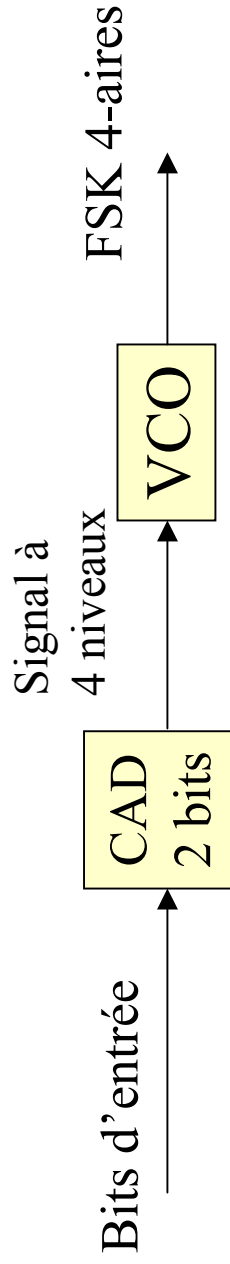
Modulateur FSK



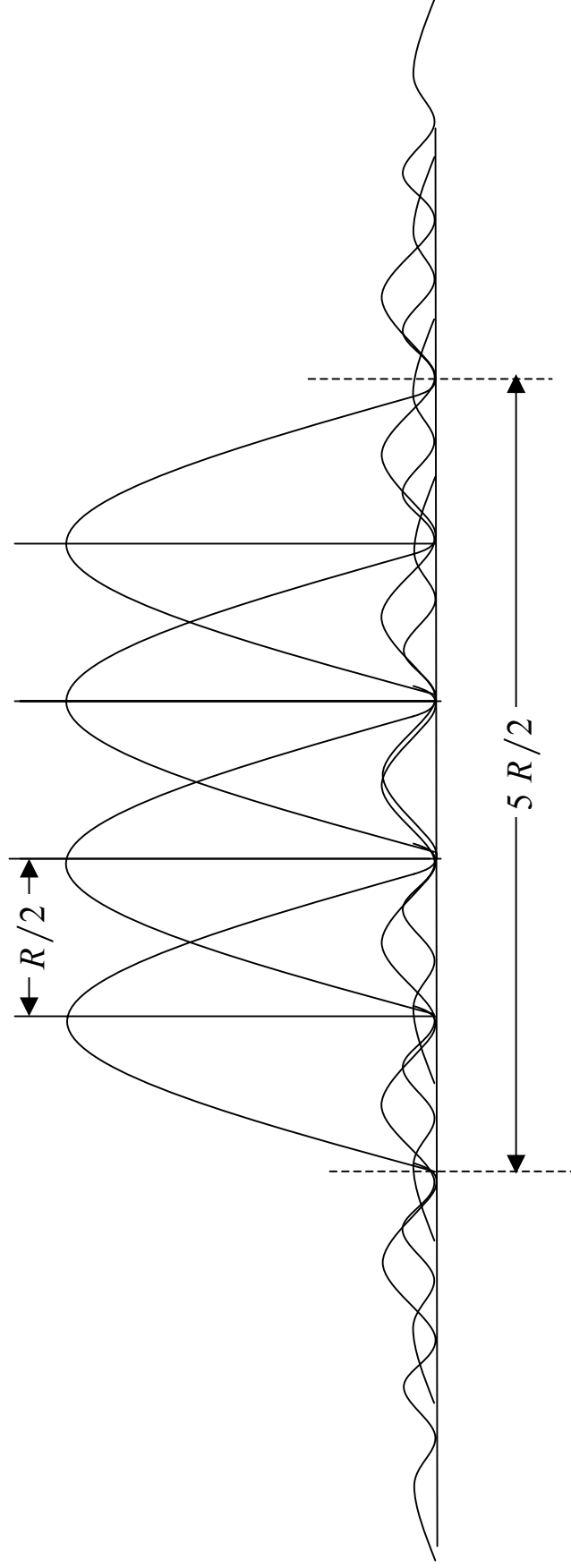
FSK-OTS (Orthogonal Tone Spacing)



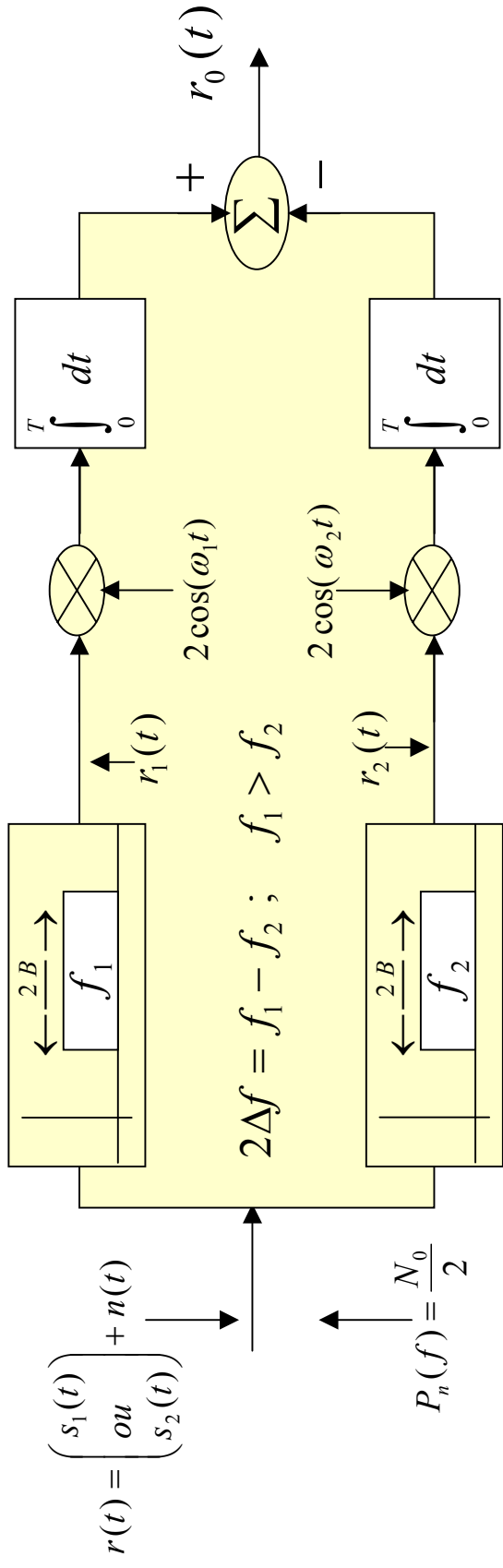
FSK 4-aires



Modulateur FSK 4-aires



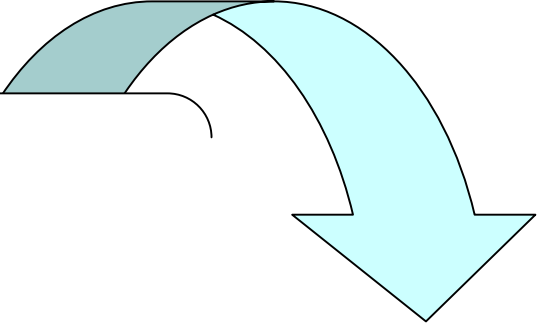
Détection cohérente par filtre adapté d'un signal FSK



$$\begin{aligned}
 E_d &= \int_0^T [A \cos(\omega_1 t) - A \cos(\omega_2 t)]^2 dt \\
 &= \int_0^T A^2 \cos^2(\omega_1 t) + A^2 \cos^2(\omega_2 t) - 2A^2 \cos(\omega_1 t) \cos(\omega_2 t) dt \\
 &= \frac{A^2 T}{2} + \frac{A^2 T}{2} - \underbrace{A^2 \int_0^T \cos(\omega_1 - \omega_2)t dt - A^2 \int_0^T \cos(\omega_1 + \omega_2)t dt}_{\ll \frac{A^2 T}{2}}
 \end{aligned}$$

(Lemme de Riemann-Lebesgue)

Détection cohérente par filtre adapté d'un signal FSK

$$\left\{ \begin{array}{l} 2\Delta f = f_1 - f_2 = n\frac{T}{2} = nR/2 \rightarrow s_1(t) \text{ et } s_2(t) \text{ sont orthogonaux} \\ E_d = \frac{A^2T}{2} + \frac{A^2T}{2} - A^2 \int_0^T \cos(\omega_1 - \omega_2)t dt \stackrel{OTS}{=} A^2T \\ E_b = \frac{E_1 + E_0}{2} = \frac{A^2T/2 + A^2T/2}{2} = \frac{A^2T}{2} \end{array} \right.$$

$$P_e = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$